

# Fick's Law

## Validity:

1. The medium is infinite. Integration over all space.

$e^{-\Sigma_t r}$  ► after few mean free paths ► 0 ►

corrections at the surface are still required.

2. The medium is uniform.  $\Sigma_s$  not  $\Sigma_s(\vec{r})$

$\Sigma_s(\vec{r})$  ►  $\phi$  and  $\Sigma$  are functions of space ► re-

derivation of Fick's law? ► locally larger  $\Sigma_s$  ► extra

$\mathcal{J}$  cancelled by  $e^{-\Sigma_t r} = e^{-(\Sigma_a + \Sigma_s)r}$  iff ???

**HW 15**

Note: assumption 5 is also violated!

3. There are no neutron sources in the medium.

Again, sources are few mean free paths away and corrections otherwise.

# Fick's Law

4. Scattering is isotropic in the lab. coordinate system!

If  $\bar{\mu} = \overline{\cos(\theta)} = \frac{2}{3A} \neq 0$  ► reevaluate  $D$ . **HW 16**

$$D = \frac{1}{3(\Sigma_t - \Sigma_s \bar{\mu})} = \frac{1}{3\Sigma_{tr}} = \frac{\lambda_{tr}}{3}$$

Weekly absorbing  $\Sigma_t = \Sigma_s$ .

For “practical” moderators:

$$\lambda_{tr} \cong \frac{\lambda_s}{1 - \bar{\mu}}$$

5. The flux is a slowly varying function of position.

$\Sigma_a \uparrow$  ► variation in  $\phi \uparrow$ .

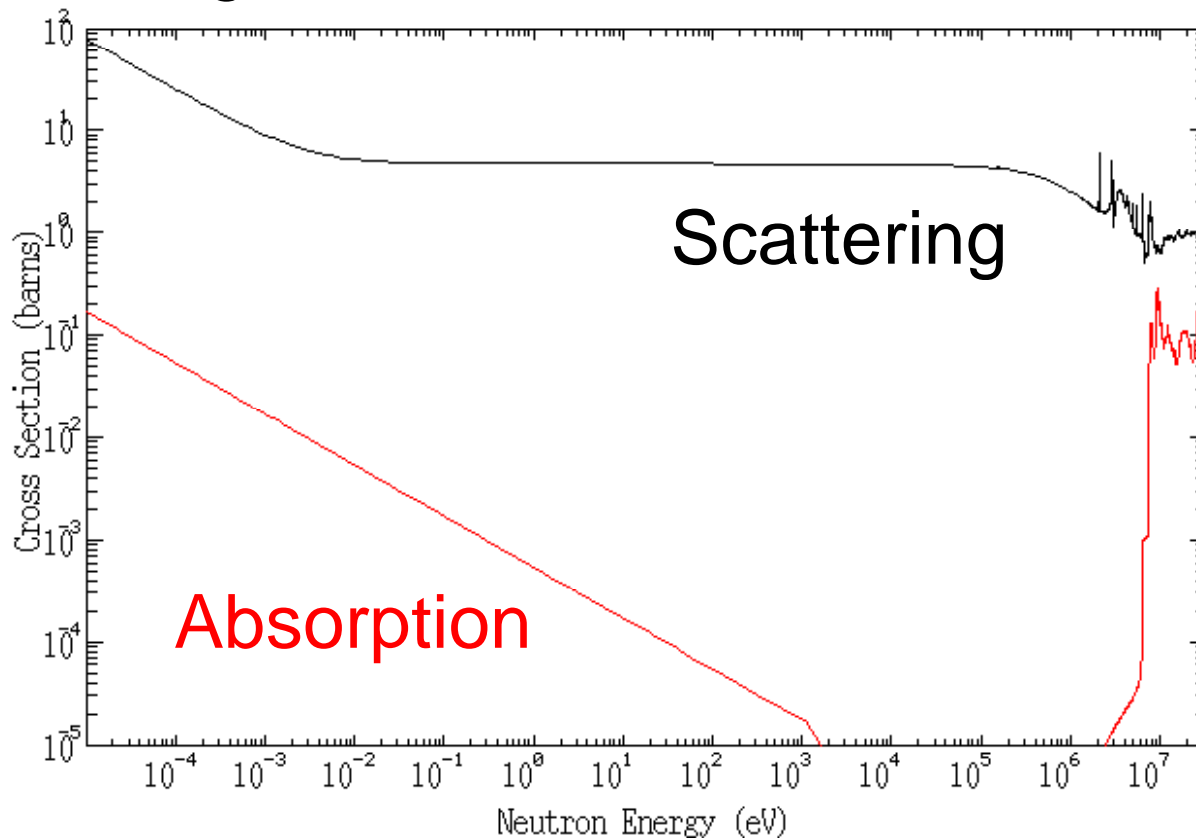
$$\frac{\partial^2 \phi}{\partial r^2}(\vec{r}) \quad ?$$

# Fick's Law

## HW 17

Estimate the diffusion coefficient of graphite at 1 eV.

The scattering cross section of carbon at 1 eV is 4.8 b.



Other  
materials?

# Fick's Law

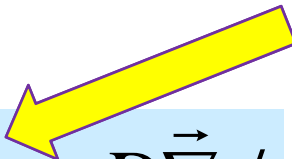
6. The neutron flux is not a function of time.

Time needed for a thermal neutron to traverse 3 mean free paths  $\sim 1 \times 10^{-3} \text{ s}$  (How?).

If flux changes by 10% per second!!!!!!

$$\left. \frac{\Delta\phi}{\phi} \right|_{1ms} = \frac{\Delta\phi / \phi}{1s} 1ms = 0.1 \times 10^{-3} = 1 \times 10^{-4}$$

Very small fractional change during the time needed for the neutron to travel this “significant” distance.


$$J \cong -D\vec{\nabla}\phi$$

# Back to the Continuity Equation

$$\frac{1}{v} \frac{\partial}{\partial t} \phi(\vec{r}, t) = S(\vec{r}, t) - \sum_a(\vec{r}) \phi(\vec{r}, t) - \vec{\nabla} \cdot \vec{J}(\vec{r}, t)$$



$$\frac{1}{v} \frac{\partial}{\partial t} \phi(\vec{r}, t) = S(\vec{r}, t) - \sum_a(\vec{r}) \phi(\vec{r}, t) + \vec{\nabla} \cdot D \vec{\nabla} \phi(\vec{r}, t)$$



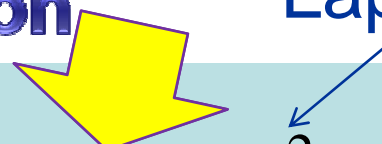
# The Diffusion Equation

$$\frac{1}{v} \frac{\partial}{\partial t} \phi(\vec{r}, t) = S(\vec{r}, t) - \sum_a (\vec{r}) \phi(\vec{r}, t) + \vec{\nabla} \cdot D \vec{\nabla} \phi(\vec{r}, t)$$

If  $D$  is independent of  $\mathbf{r}$  (uniform medium)

**The Diffusion Equation**

Laplacian

$$\frac{1}{v} \frac{\partial}{\partial t} \phi(\vec{r}, t) = S(\vec{r}, t) - \sum_a (\vec{r}) \phi(\vec{r}, t) + D \nabla^2 \phi(\vec{r}, t)$$


**The Steady State Diffusion Equation** or scalar Helmholtz equation.

$$0 = S(\vec{r}) - \sum_a (\vec{r}) \phi(\vec{r}) + D \nabla^2 \phi(\vec{r})$$

**Non-multiplying medium (and steady state)**

$$0 = - \sum_a (\vec{r}) \phi(\vec{r}) + D \nabla^2 \phi(\vec{r})$$

Buckling equation.

# Steady State Diffusion Equation

$$0 = S(\vec{r}) - \Sigma_a(\vec{r})\phi(\vec{r}) + D\nabla^2\phi(\vec{r})$$

Define  $L^2 = \frac{D}{\Sigma_a}$

$L \equiv$  Diffusion Length  
 $L^2 \equiv$  Diffusion Area  
**Moderation Length**

$$\nabla^2\phi - \frac{1}{L^2}\phi = -\frac{S}{D}$$

$$\nabla^2\phi - \frac{1}{L^2}\phi = 0$$

**Non-multiplying  
medium**

## Boundary Conditions

- Solve DE ► get  $\phi$ .
- Solution must satisfy BC's.
- Solution should be real and non-negative.

# Steady State Diffusion Equation

## One-speed neutron diffusion in infinite medium

Point source

$$\nabla^2 \phi(\vec{r}) - \frac{1}{L^2} \phi(\vec{r}) = 0$$



**HW 18**

$$\frac{d^2}{dr^2} \phi(r) + \frac{2}{r} \frac{d}{dr} \phi(r) - \frac{1}{L^2} \phi(r) = 0$$

General solution

$$\phi = A \frac{e^{-r/L}}{r} + C \frac{e^{r/L}}{r}$$

$A, C$  determined from BC's.



# Steady State Diffusion Equation

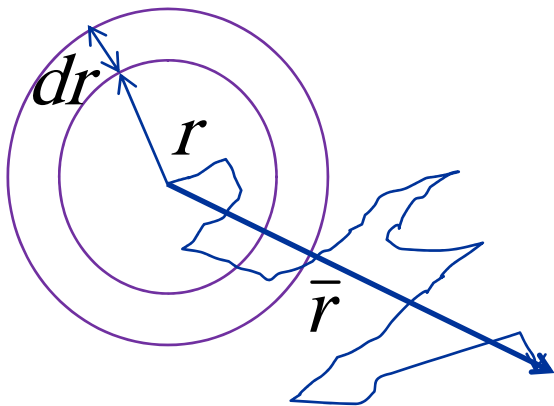
**BC**  $r \rightarrow \infty \rightarrow \phi \rightarrow 0 \rightarrow C = 0.$

**HW 18 (continued)**

$$\phi = A \frac{e^{-r/L}}{r}$$

Show that  $A = \frac{S}{4\pi D} \rightarrow \phi = \frac{S}{4\pi D} \frac{e^{-r/L}}{r} \quad L^2 = \frac{D}{\Sigma_a}$

$4\pi r^2 dr \Sigma_a \phi$  neutrons per second absorbed in the ring.



Show that

$$\overline{r^2} = 6L^2$$

# Steady State Diffusion Equation

Scalar flux, vector current.

## HW 19

Study example 5.3 and solve problem 5.8 in Lamarsh.

Multiple Point Sources?

# Steady State Diffusion Equation

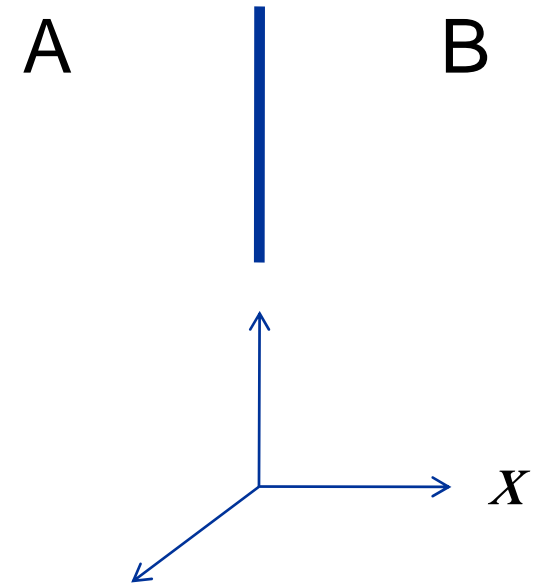
## One-speed neutron diffusion in a finite medium

- At the interface

$$\phi_A = \phi_B$$

$$J_A = J_B \Rightarrow -D_A \frac{d\phi_A}{dx} = -D_B \frac{d\phi_B}{dx}$$

- What if  $A$  or  $B$  is a vacuum?
- Linear extrapolation distance.



# More realistic multiplying medium

## One-speed neutron diffusion in a multiplying medium

The reactor core is a finite multiplying medium.

- Neutron flux?
- Reaction rates?
- Power distribution in the reactor core?

Recall:

- **Critical (or steady-state):**

**Number of neutrons produced by fission = number of neutrons lost by:**

**absorption**

**and**

**leakage**

$$k_{\infty} = \frac{\text{neutron production rate } (S)}{\text{neutron absorption rate } (A)}$$

$$k_{eff} = \frac{\text{neutron production rate } (S)}{\text{neutron absorption rate } (A) + \text{neutron leakage rate } (LE)}$$

# More realistic multiplying medium

$$\frac{k_{eff}}{k_{\infty}} = \frac{A}{A + LE} = P_{non-leak}$$

Things to be used later...!

non - leakage probability

$$LE \propto SA \quad \text{surface area}$$

$$S \propto V \quad \text{Volume}$$

$$\frac{LE}{S} \propto \frac{SA}{V} \propto \frac{a^2}{a^3} = \frac{1}{a}$$

Recall:

For a critical reactor:

$$K_{eff} = 1$$

$$K_{\infty} > 1$$

## Steady state homogeneous reactor

$$0 = \sum_a k_{\infty} \phi(\vec{r}) - \sum_a \phi(\vec{r}) + D \nabla^2 \phi(\vec{r})$$

$$\nabla^2 \phi(\vec{r}) + B^2 \phi(\vec{r}) = 0$$

$$B^2 \equiv \frac{k_{\infty} - 1}{L^2}$$

multiplying medium

# More realistic multiplying medium

$$\nabla^2 \phi(\vec{r}) + B^2 \phi(\vec{r}) = 0$$

$$B^2 = -\frac{\nabla^2 \phi(\vec{r})}{\phi(\vec{r})}$$

- The buckling is a measure of extent to which the flux curves or “buckles.”
- For a slab reactor, the buckling goes to zero as “ $a$ ” goes to infinity. There would be no buckling or curvature in a reactor of infinite width.
- Buckling can be used to infer leakage. The greater the curvature, the more leakage would be expected.

# More on One-Speed Diffusion

## HW 20



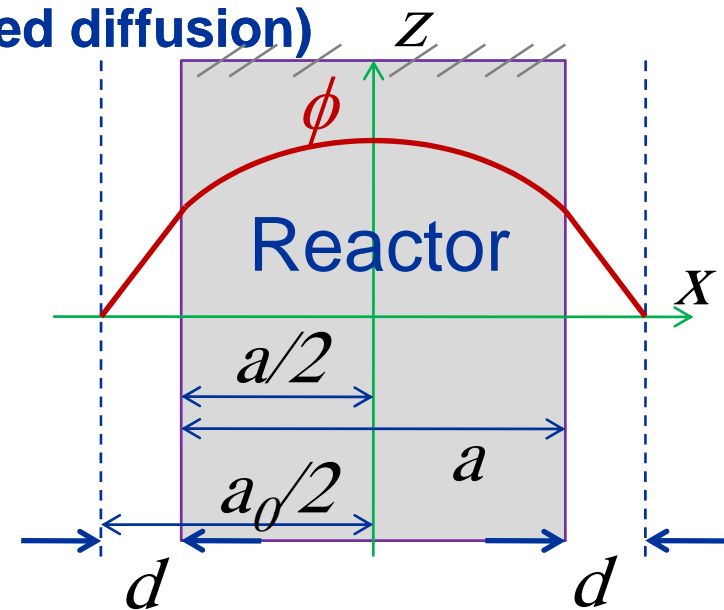
Show that for a **critical homogeneous reactor**

$$P_{non-leak} = \frac{1}{B^2 L^2 + 1} = \frac{\sum_a \phi}{\sum_a \phi - D \nabla^2 \phi} = \frac{\sum_a \phi}{\sum_a \phi + B^2 D \phi}$$

## Infinite Bare Slab Reactor (one-speed diffusion)

- Vacuum beyond.
- Return current = 0.

$d$  = linear extrapolation distance  
=  $0.71 \lambda_{tr}$  (for plane surfaces)  
=  $2.13 D$ .



# More on One-Speed Diffusion

## HW 21

For the infinite slab  $\frac{d^2\phi}{dx^2} + B^2\phi = 0$ . Show that the general solution

$$\phi(x) = A \cos Bx + C \sin Bx$$

with BC's

$$\phi\left(\pm \frac{a_0}{2}\right) = 0$$

$$\left. \frac{d\phi(x)}{dx} \right|_{x=0} = 0$$

Flux is symmetric about the origin.


$$\phi(x) = A \cos Bx \quad A = \phi_0$$


$$\phi\left(\pm \frac{a_0}{2}\right) = A \cos B\left(\pm \frac{a_0}{2}\right) = 0 \Rightarrow B\left(\pm \frac{a_0}{2}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$



# More on One-Speed Diffusion

## HW 21 (continued)

$$B(\pm \frac{a_0}{2}) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$a_0 = \frac{\pi}{B}, \frac{3\pi}{B}, \frac{5\pi}{B}, \dots$$


Fundamental mode, the only mode significant in critical reactors.

$$\phi(x) = \phi_0 \cos \frac{\pi}{a_0} x \quad B^2 = \left( \frac{\pi}{a_0} \right)^2 \equiv \text{Geometrical Buckling}$$

For a critical reactor, the geometrical buckling is equal to the material buckling.

To achieve criticality



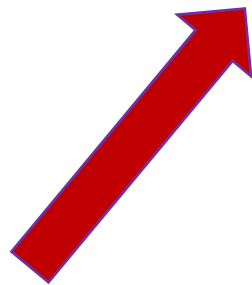
$$\left( \frac{\pi}{a_0} \right)^2 = \frac{k_\infty - 1}{L^2}$$

# More on One-Speed Diffusion

$\phi_0$  ???

- To achieve criticality  $\left(\frac{\pi}{a_0}\right)^2 = \frac{k_\infty - 1}{L^2}$
- But criticality at what power level??
- $\phi_0$  can not be determined by the geometry alone.

$$\phi(x) = \phi_0(P, \dots) \cos \frac{\pi}{a_0} x$$



**Do it.**