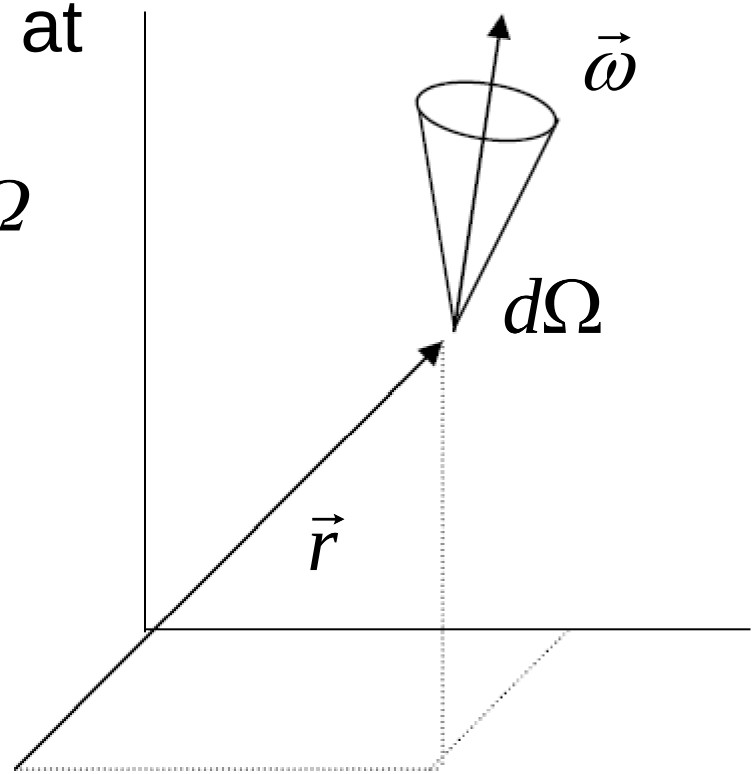
One-Speed Interactions

$n(\vec{r},\vec{\omega})d\Omega \equiv$ Neutrons per cm^3 at $\boldsymbol{r}$ whose velocity vector lies within $d\Omega$ about $\boldsymbol{\omega}$.



$$n(\vec{r}) = \int_{4\pi} n(\vec{r},\vec{\omega})d\Omega$$

- Same argument as before ▶

$$dI(\vec{r},\vec{\omega}) = n(\vec{r},\vec{\omega})v\,d\Omega$$

$$dF(\vec{r},\vec{\omega}) = \Sigma_t(\vec{r})dI(\vec{r},\vec{\omega})$$

Scalar

$$R(\vec{r}) = F(\vec{r}) = \int_{\omega} dF(\vec{r},\vec{\omega}) = \Sigma_t(\vec{r})v\int_{4\pi} n(\vec{r},\vec{\omega})d\Omega = \Sigma_t(\vec{r})vn(\vec{r}) = \Sigma_t(\vec{r})\phi(\vec{r})$$

where $\phi(\vec{r}) = v\int_{4\pi} n(\vec{r},\vec{\omega})d\Omega$

Multiple Energy Interactions

- Generalize to include energy

$n(\vec{r}, E, \vec{\omega})dEd\Omega$ ≡ Neutrons per cm^3 at $\boldsymbol{r}$ with energy interval (E, $E+dE$) whose velocity vector lies within $d\Omega$ about $\boldsymbol{\omega}$.

$$n(\vec{r}, E)dE = \int_{4\pi} n(\vec{r}, E, \vec{\omega})d\Omega dE \qquad n(\vec{r}) = \int_0^{\infty}\int_{4\pi} n(\vec{r}, E, \vec{\omega})d\Omega dE$$

$$R_t(\vec{r}, E)dE = \Sigma_t(\vec{r}, E)n(\vec{r}, E)v(E)dE = \Sigma_t(\vec{r}, E)\phi(\vec{r}, E)dE$$

$$R_t(\vec{r}) = \int_0^{\infty} \Sigma_t(\vec{r}, E)\phi(\vec{r}, E)dE$$

Scalar

Thus knowing the material properties Σ_t and the neutron flux ϕ, *both* as functions of space and energy, we can calculate the interaction rate throughout the reactor.

Neutron Current

- Similarly $R_S(\vec{r}) = \int_0^{\infty} \Sigma_S(\vec{r}, E)\phi(\vec{r}, E)dE$ and so on …

Scalar

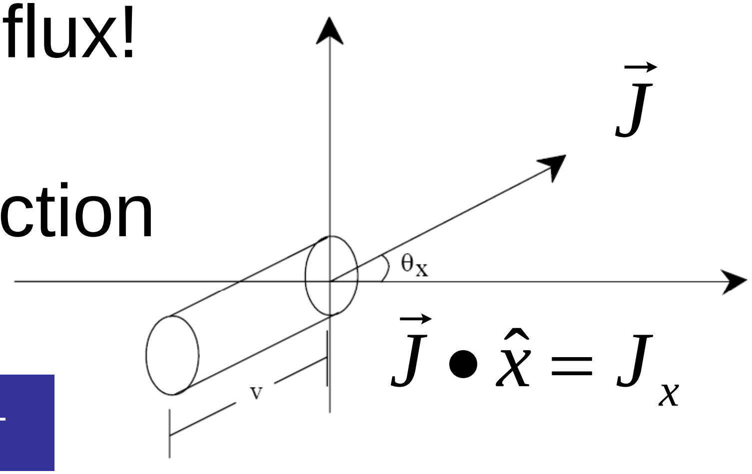
- Redefine $dI(\vec{r}, \vec{\omega}) = n(\vec{r}, \vec{\omega})vd\Omega$ as $d\vec{I}(\vec{r}, \vec{\omega}) = n(\vec{r}, \vec{\omega})\vec{v}d\Omega$

One group!

$$\phi(\vec{r}) = \int_{4\pi} vn(\vec{r}, \vec{\omega})d\Omega \quad \Rightarrow \quad \vec{J} = \int_{4\pi} \vec{v}n(\vec{r}, \vec{\omega})d\Omega$$

Neutron current density

- From larger flux to smaller flux!
- Neutrons are not pushed!
- More scattering in one direction than in the other.



$$\vec{J} \bullet \hat{x} = J_x$$

Equation of Continuity

Net flow of neutrons per second per unit area normal to the x direction:

$$\vec{J} \bullet \hat{x} = J_x = \int_{4\pi} n(\vec{r},\vec{\omega}) v \cos\theta_x d\Omega$$

In general: $\vec{J} \bullet \hat{n} = J_n$

Equation of Continuity

$$\frac{\partial}{\partial t}\int_{\forall} n(\vec{r},t)d\forall = \int_{\forall} S(\vec{r},t)d\forall - \int_{\forall} \Sigma_a(\vec{r})\phi(\vec{r},t)d\forall - \oint_A \vec{J}(\vec{r},t) \overset{\pm}{\bullet} \hat{n}dA$$

Rate of change in number of neutrons

Production rate

Absorption rate

"Leakage in/out" rate

Volume

Source distribution function

Surface area bounding $\forall$

Normal to A *(outwards)*

Equation of Continuity

Using Gauss' Divergence Theorem

$$\oint_S \vec{B} \bullet d\vec{A} = \int_V \vec{\nabla} \bullet \vec{B} d^3 r$$

Recall:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_A \vec{J}(\vec{r},t) \bullet \hat{n} dA = \int_\forall \vec{\nabla} \bullet \vec{J}(\vec{r},t) d\forall$$

$$\frac{\partial}{\partial t}\int_\forall n(\vec{r},t) d\forall = \int_\forall S(\vec{r},t) d\forall - \int_\forall \Sigma_a(\vec{r})\phi(\vec{r},t) d\forall - \oint_A \vec{J}(\vec{r},t) \bullet \hat{n} dA$$

Both flux and current!!

Convert current to flux?

One group!

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = S(\vec{r},t) - \underbrace{\Sigma_a(\vec{r})\phi(\vec{r},t) - \vec{\nabla} \bullet \vec{J}(\vec{r},t)}$$

Fundamental equation in Reactor Theory

Equation of Continuity

Equation of Continuity

Steady state

$$S(\vec{r}) - \Sigma_a(\vec{r})\phi(\vec{r}) - \vec{\nabla} \bullet \vec{J}(\vec{r}) = 0$$

Non-spacial dependence

$$\frac{\partial}{\partial t} n(t) = S(t) - \Sigma_a \phi(t)$$

Delayed sources? Will do it later.

Fick's Law

- The **exact** interpretation of neutron **transport** in heterogeneous domains is so complex.
- Assumptions and approximations.
- Simplified approaches.
- Simplified but accurate enough to give an **estimate** of the **average characteristics** of **neutron population**.
- Numerical solutions.
- Monte Carlo techniques.

MCNP
Geant4

Fick's Law

Assumptions:

1. The medium is infinite.
2. The medium is uniform $\sum not \sum(\vec{r})$.
3. There are no neutron sources in the medium.
4. Scattering is isotropic in the lab coordinate system.
5. The neutron flux is a slowly varying function of position.
6. The neutron flux is not a function of time.

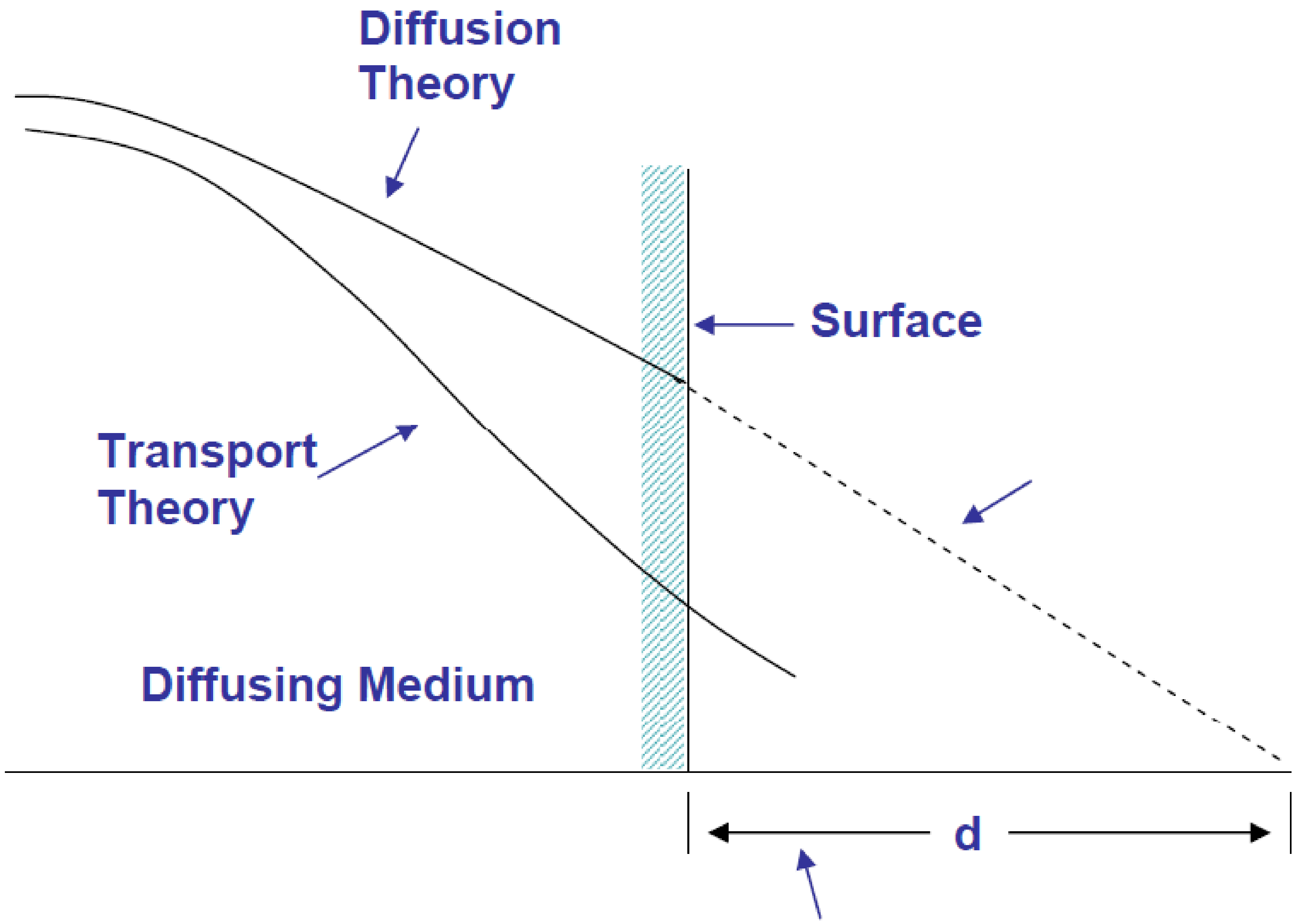
Restrictive! **Applicability??**

http://www.iop.org/EJ/article/0143-0807/26/5/023/ejp5_5_023.pdf

Fick's Law

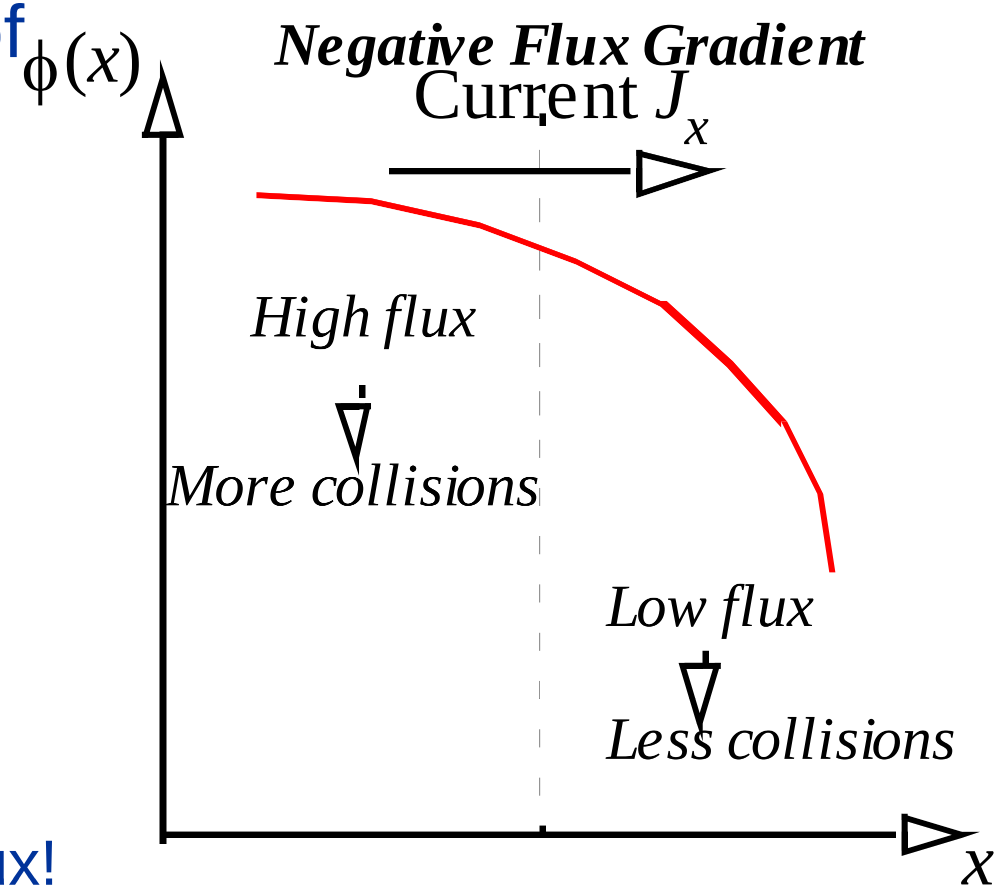
Lamarsh puts it more bluntly:
"Fick's Law is invalid:
a) in a medium that strongly absorbs neutrons;
b) within three mean free paths of either a neutron source or the surface of a material; and
c) when neutron scattering is strongly anisotropic."

Fick's Law


Diffusion
Theory
Surface
Transport
Theory
Diffusing Medium
d

Fick's Law

- Diffusion: random walk of an ensemble of particles from region of high "concentration" to region of small "concentration".
- Flow is proportional to the negative gradient of the "concentration".

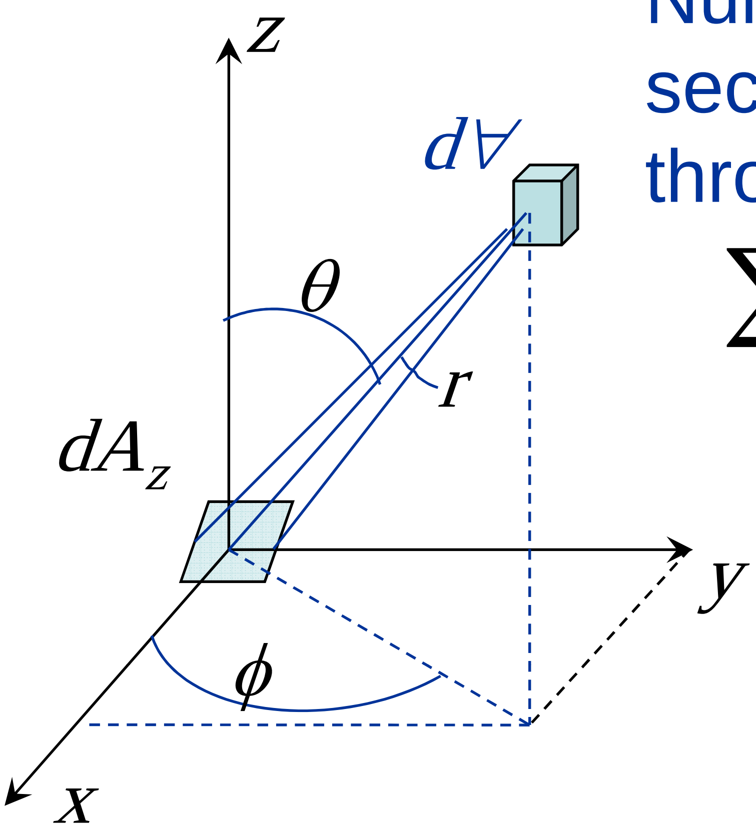


$$J_x = -D\frac{\partial \phi}{\partial x}$$

Recall:

- From larger flux to smaller flux!
- Neutrons are not pushed!
- More scattering in one direction than in the other.

Fick's Law



Number of neutrons **scattered** per second from $d\forall$ at $\mathbf{r}$ and going through dA_z

$$\Sigma_s \, \phi(\vec{r}) \frac{\cos\theta dA_z}{4\pi r^2} e^{-\Sigma_t r} d\forall$$

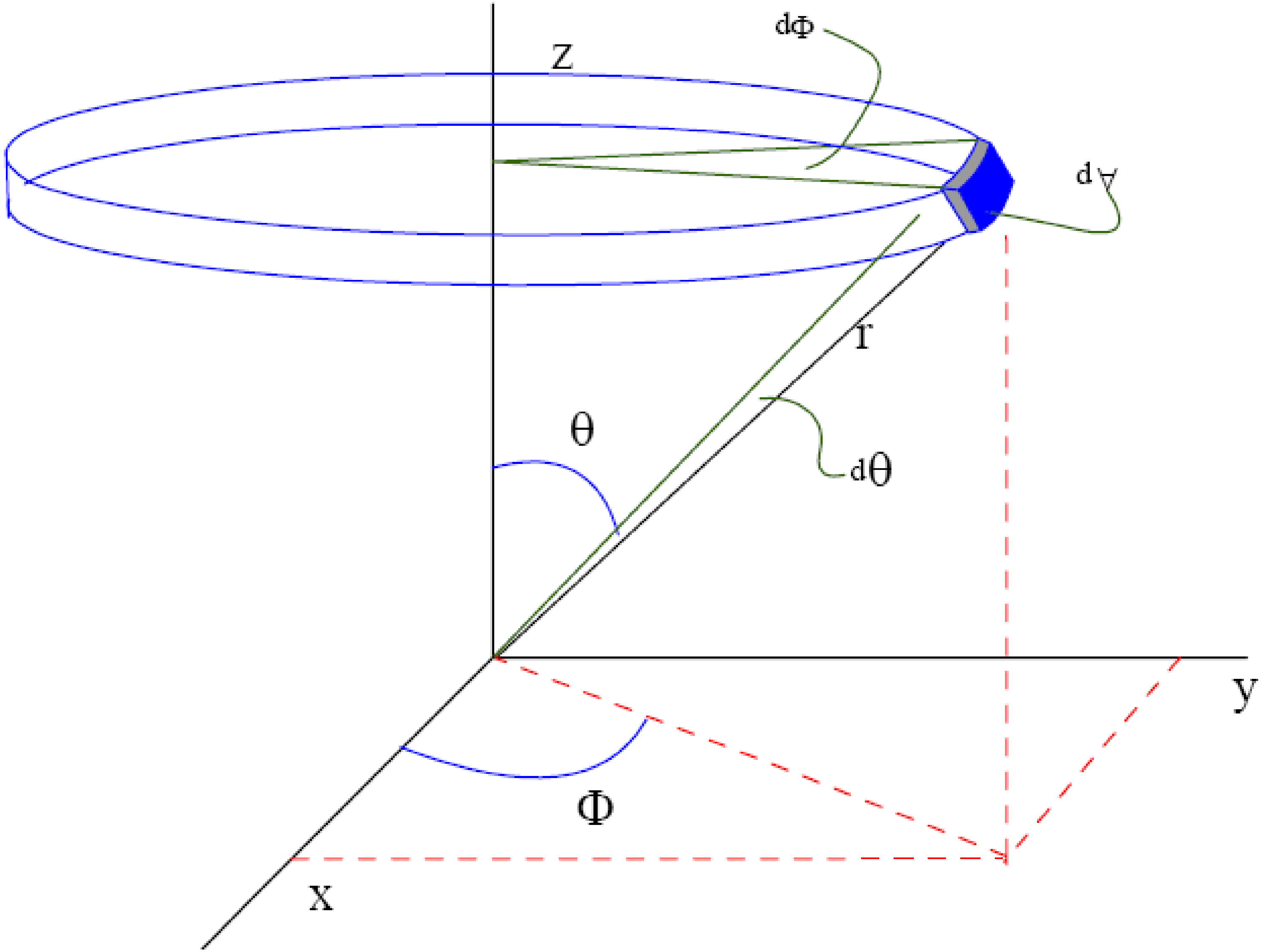
Σ_s not $\Sigma_s(\vec{r})$

Slowly varying

Isotropic

Removed *en route* (assuming no buildup)

Fick's Law



Fick's Law

HW 14

$$J_z^- dA_z = \frac{\Sigma_s dA_z}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} \phi(\vec{r}) e^{-\Sigma_t r} \left[\cos\theta \sin\theta dr d\theta d\phi\right]$$

$$J_z^+ dA_z = ?$$

and show that $J_z = J_z^+ - J_z^- = -\left(\frac{\Sigma_s}{3\Sigma_t^2}\right)\left(\frac{\partial \phi}{\partial z}\right)_0$

$D \approx \frac{1}{3\Sigma_s}$?

Fick's law

and generalize $J = -D\vec{\nabla}\phi$

Diffusion coefficient $D = \frac{\Sigma_s}{3\Sigma_t^2}$ → Total removal

The current density is proportional to the negative of the gradient of the neutron flux.

Fick's Law

Validity:

1. The medium is infinite. Integration over all space.

   $e^{-\Sigma_t r}$ ▶ after few mean free paths ▶ 0 ▶ corrections at the surface are still required.

2. The medium is uniform. $\Sigma_s \; not \; \Sigma_s(\vec{r})$

   $\Sigma_s(\vec{r})$ ▶ ϕ and Σ are functions of space ▶ re-derivation of Fick's law? ▶ locally larger Σ_s ▶ extra ***J*** cancelled by $e^{-\Sigma_t r} = e^{-(\Sigma_a + \Sigma_s) r}$ iff ??? **HW 15**

   Note: assumption 5 is also violated!

3. There are no neutron sources in the medium.

   Again, sources are few mean free paths away and corrections otherwise.