Controlled Fission

$$k_\infty = fp\varepsilon\eta, \qquad k_{eff} = f\rho\varepsilon\eta P_{non-leak}$$

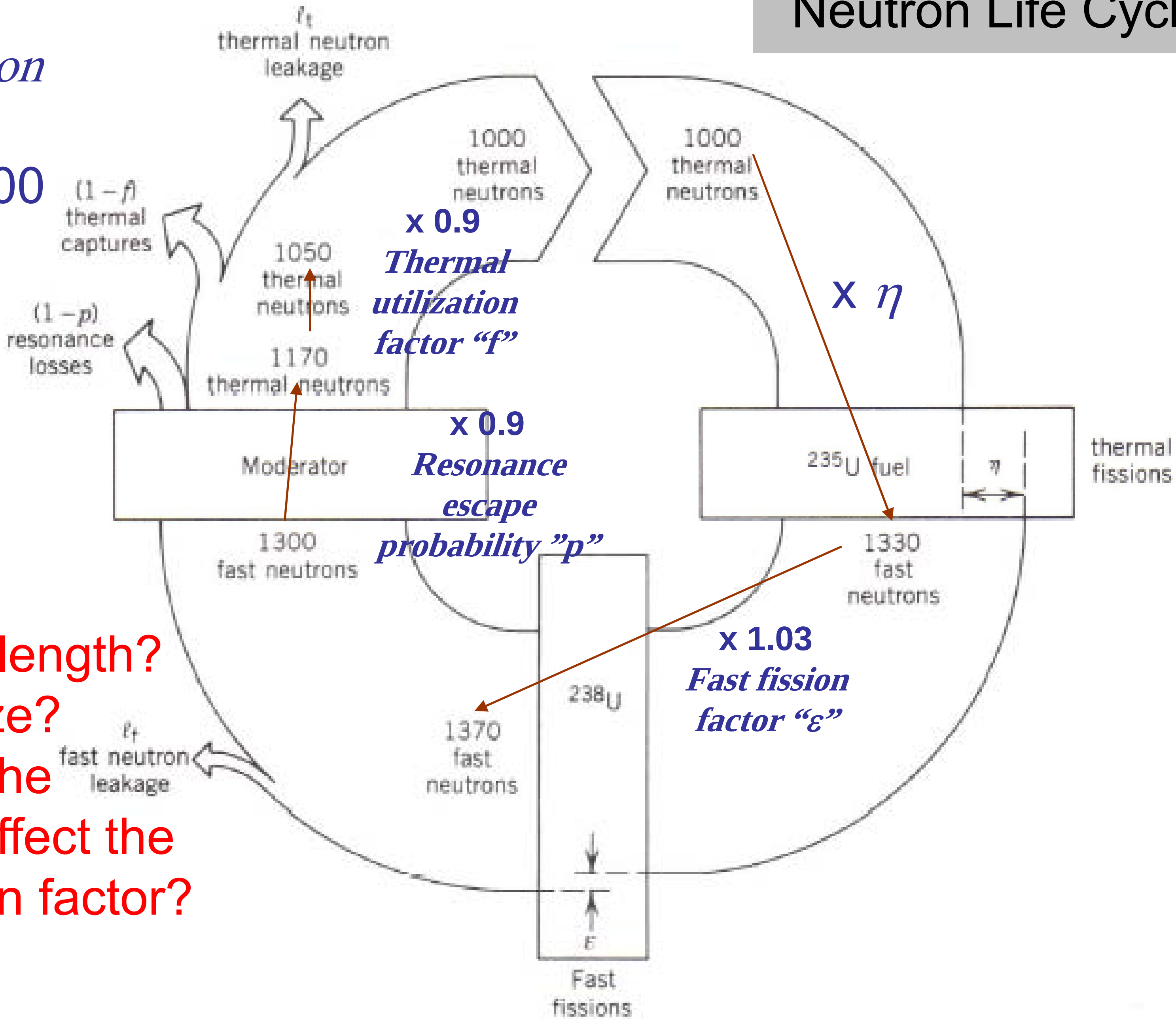
- Fast from thermal, $\eta = \frac{1}{\Sigma_a}\sum_i \nu(i)\Sigma_f(i)$ as defined in HW 11.
- Fast from fast, ε.
- Thermal from fast, p.
- Thermal available for fuel $f = \frac{\sum_a^{fuel}}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator} + \sum_a^{rods} + \sum_a^{poison} + ..}$

Thinking QUIZ

- For each thermal neutron absorbed, how many fast neutrons are produced? Will need this when discuss two-group diffusion.

Neutron reproduction factor

$k_{eff} = 1.000$

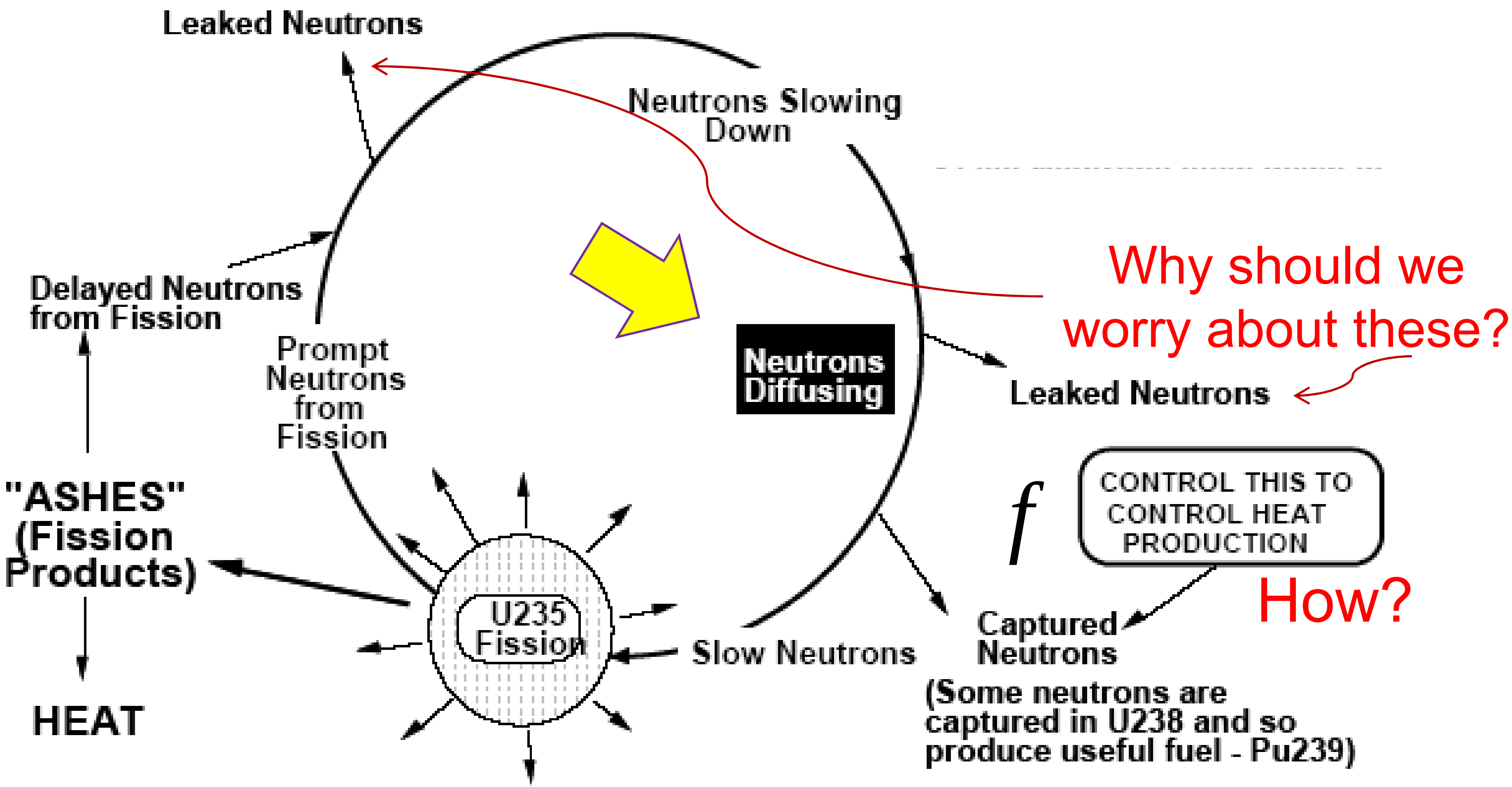


What is:

- Migration length?
- Critical size?

How does the geometry affect the reproduction factor?

Neutron Life Cycle



Controlled Fission

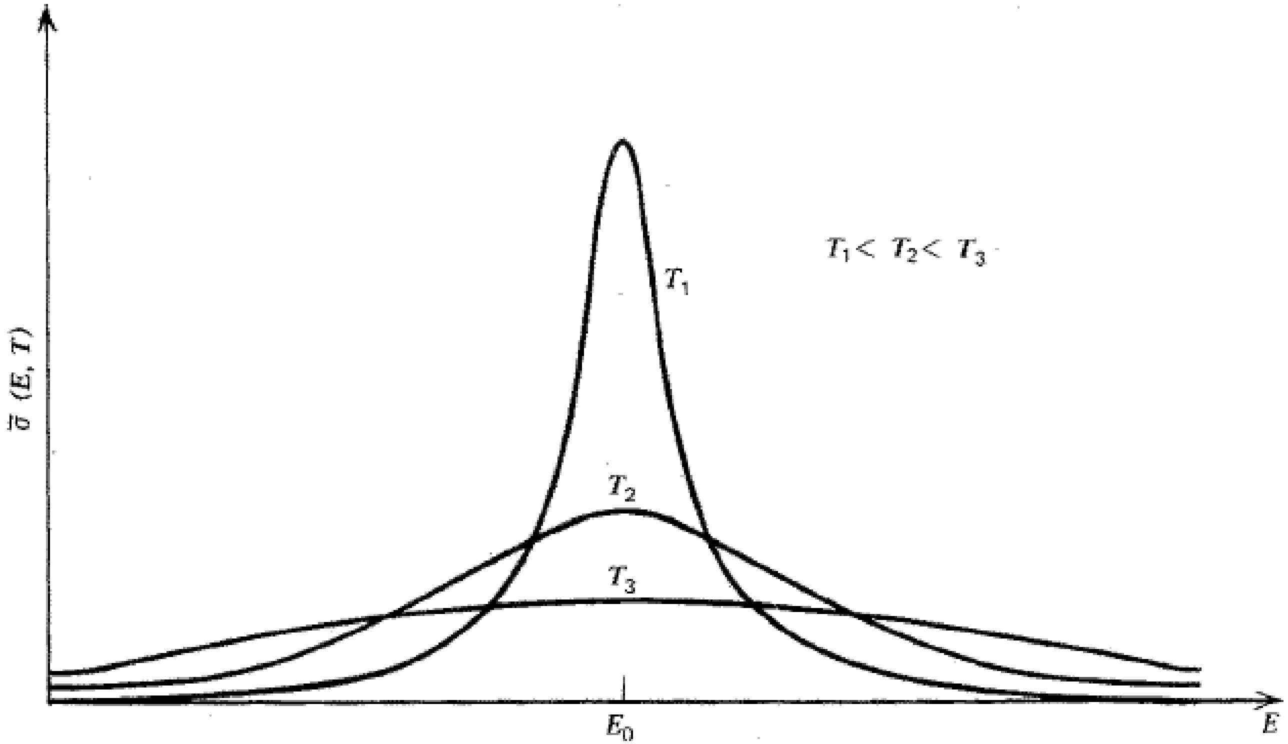
$$k = fp\varepsilon\eta(1-l_{fast})(1-l_{thermal})$$ Not fixed…!

- Thermal utilization factor f can be changed, as an example, by adding absorber to coolant (PWR) (chemical shim, boric acid), or
by inserting movable control rods in & out. Poison.
- Reactors can also be controlled by altering neutron leakages using movable neutron reflectors.
- f *and* p factors change as fuel is burned.
- f, p, η change as fertile material is converted to fissile material.

Controlled Fission

- Attention should be paid also to the fact that reactor power changes occur due to changes in resonance escape probability p. If Fuel T↑, p↓ *due to* Doppler broadening of resonance peaks.

Under-moderation
and
over-moderation.



Controlled Fission

Time scale for neutron multiplication

- Time constant τ includes moderation time (~10^{-6} s) and diffusion time of thermal neutrons (~10^{-3} s).

Time	Average number of thermal neutrons
t	n
$t+\tau$	kn
$t+2\tau$	k^2n

- For a short time dt

$$\frac{dn}{dt} = \frac{kn - n}{\tau}$$

- **Show that**

$$n(t) = n_0 e^{(k-1)t/\tau}$$

Controlled Fission

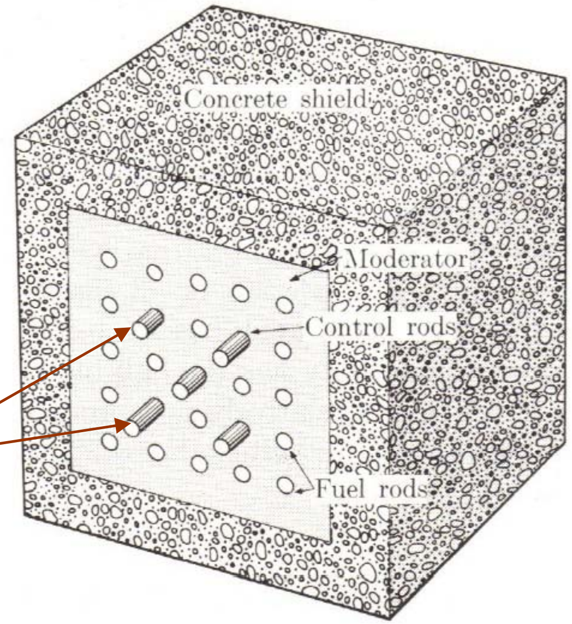
$$n(t) = n_0 e^{(k-1)t/\tau}$$

- $k = 1$ ▶ n is constant (Desired).
- $k < 1$ ▶ n decays exponentially.
- $k > 1$ ▶ n grows exponentially with time constant $\tau / (k-1)$.
- $k = 1.01$ (slightly supercritical..!) ▶ $e^{(0.01/0.001)t} = e^{10} = 22026$ **in 1s.**
- Design the reactor to be slightly subcritical for prompt neutrons.
- The "few" "delayed" neutrons will be used to achieve criticality, allowing enough time to manipulate the control rods (or use shim or …).

Reactivity.

Dangerous

Cd control rods



Fission Reactors

Essential elements:

- Fuel [fissile (or fissionable) material].
- Moderator (not in reactors using fast neutrons).

} Core

- Reflector (to reduce leakage and critical size).
- Containment vessel (to prevent leakage of waste).
- Shielding (for neutrons and γ's).
- Coolant.
- Control system.
- Emergency systems (to prevent runaway during failure).

Chapter 4 in Lamarsh

Fission Reactors

<u>**Types of reactors:**</u>

Used for what?

- Power reactors: extract kinetic energy of fragments as heat ▶ boil water ▶ steam drives turbine ▶ electricity.
- Research reactors: low power (1-10 MW) to generate neutrons (~10^{13} n.cm^{-2}.s^{-1} or higher) for research.
- Converters and breeders: Convert non-thermally-fissionable material (non-fissile) to a thermally-fissionable material (fissile).
- ADS.
- Fusion.

What are neutron generators?

Fission Reactors

What neutron energy?

- Thermal, fast reactors.
- Large, smaller but more fuel.

What fuel?

- Natural uranium, enriched uranium, ^{233}U, ^{239}Pu, Mixtures.

How???

From converter or breeder reactor.

Fission Reactors

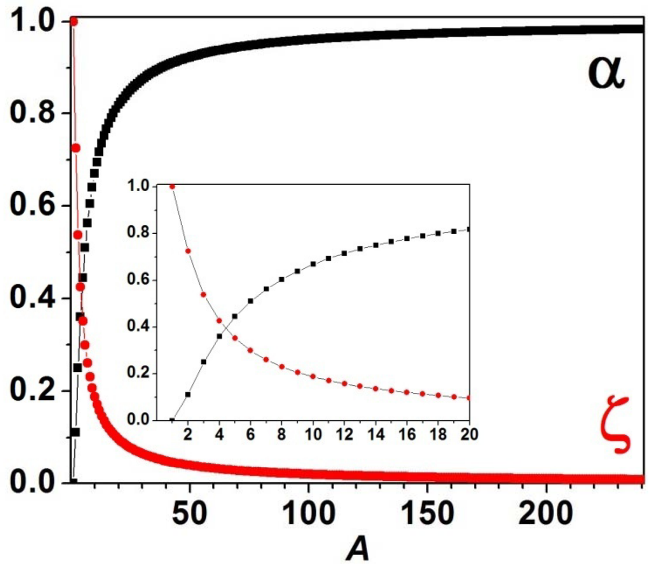
What assembly?

- Heterogeneous: moderator and fuel are lumped.
- Homogeneous: moderator and fuel are mixed together.
- In homogeneous systems, it is easier to calculate p and f for example, but a homogeneous natural uranium-graphite mixture (for example) can not go critical. Why?

What coolant?

- Coolant prevents meltdown of the core.
- It transfers heat in power reactors.
- Why pressurized-water reactors.
- Why liquid sodium?

More on Moderators



What moderator?

1. Cheap and abundant.
2. Chemically stable.
3. Low mass (high ζ *logarithmic energy decrement*).
4. High density.
5. High Σ_s and very low Σ_a.

- Graphite (1,2,4,5) increase amount to compensate 3.
- Water (1,2,3,4) but $n + p \rightarrow d + \gamma$ ▶ enriched uranium.
- D_2O (heavy water) (1!) but has low capture cross section ▶ natural uranium, but if capture occurs, produces tritium (more than a LWR).
- …..

More on Moderators

For a compound?

Moderating power

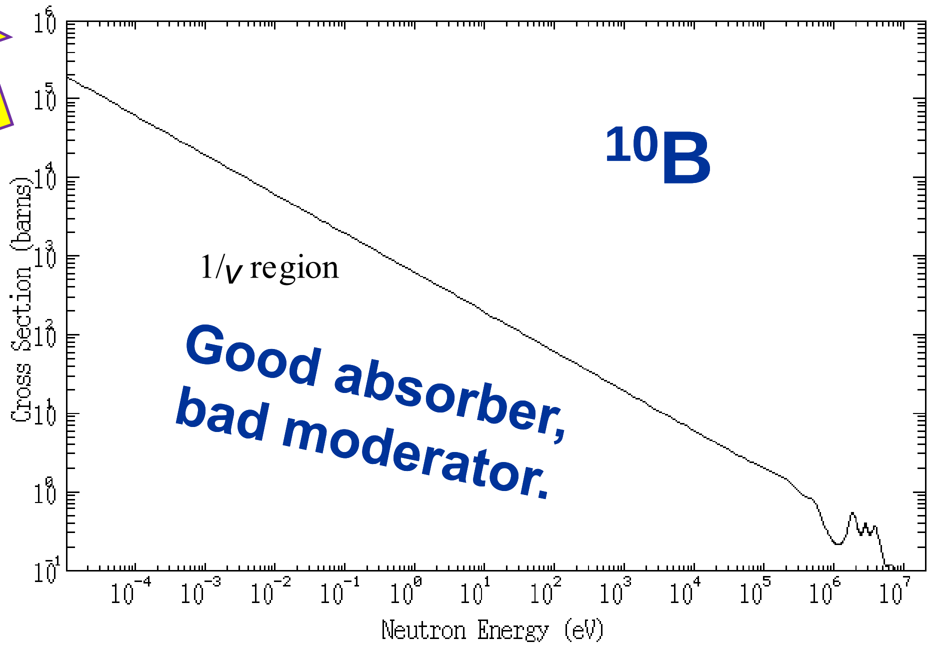
Never consider this only!

$$\text{Moderating ratio} \equiv \frac{\xi \Sigma_s}{\Sigma_a}$$

HW 12

Calculate both moderating power and ratio for water, heavy water, graphite, polyethylene and **boron**. Tabulate your results and comment.

$$^{10}B + n \rightarrow {}^{11}B^* \rightarrow {}^{7}Li + \alpha$$



More on Moderators

HW 12 (continued)

Calculate the moderating power and ratio for pure D_2O as well as for D_2O contaminated with a) 0.25% and b) 1% H_2O.
Comment on the results.
In CANDU systems there is a need for heavy water upgradors.

More on Moderators

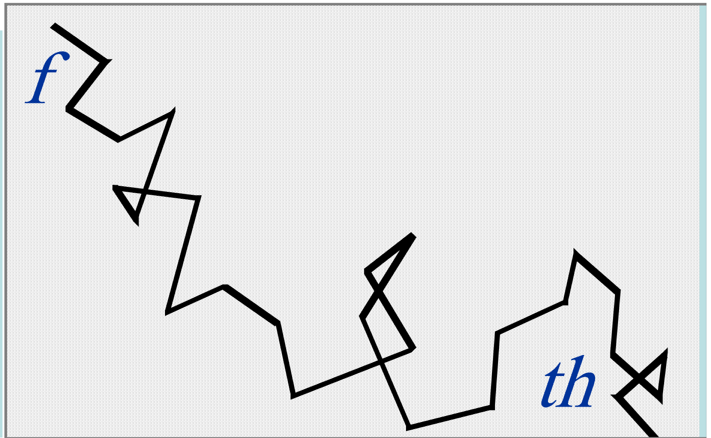
Recall $\ln E_n^{\backslash} = \ln E - n\zeta$ ▶ $n = \dfrac{\ln(E / E_n^{\backslash})}{\zeta}$

After n collisions

After one collision

$$\overline{\Delta u} = \zeta = \left[\ln \frac{E}{E^{\backslash}}\right]_{av} = 1 + \frac{(A-1)^2}{2A} \ln \frac{A-1}{A+1}$$

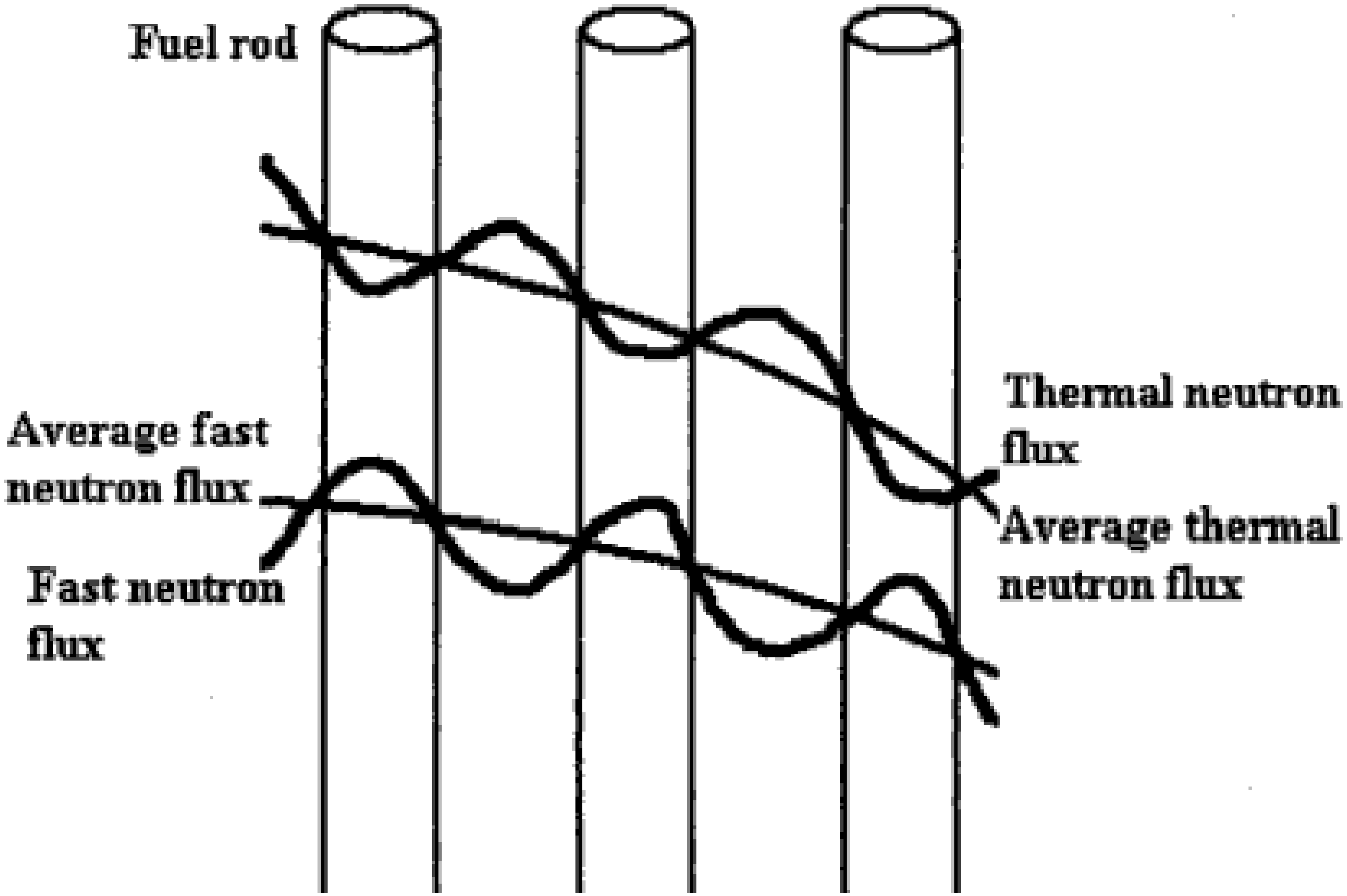
$$n = \frac{\ln(E_f / E_{th})}{\zeta}$$



Total mean free path = $n\,\lambda_s$

Is it random walk or there is a preferred direction???

More on Moderators



More on Moderators

After one collision.

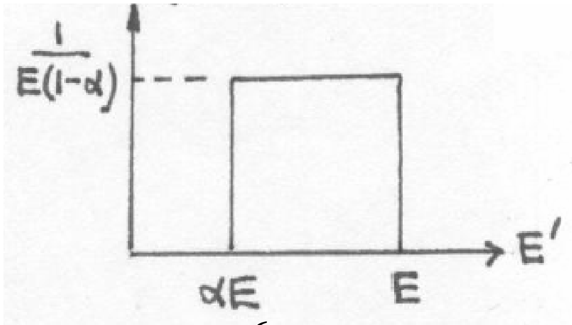
Recall $E'_{min} = \left(\frac{A-1}{A+1}\right)^2 E \equiv \alpha E$ (head-on). Then the maximum energy loss is $(1-\alpha)E$, or $\alpha E \leq E' \leq E$.

For an ***s*-wave** collision:

$$\int_{\alpha E}^{E} P(E \to E')dE' = 1 \qquad \therefore \qquad P(E \to E') = \frac{1}{(1-\alpha)E}$$

Flat-top probability

$$\overline{E'} = \tfrac{1}{2}(1+\alpha)E$$



Assumptions:

1. **Elastic scattering.** $E\downarrow$
2. **Target nucleus at rest.** $E\uparrow$
3. **Spherical symmetry in CM.**

Obviously $\sigma_s(E \to E') \equiv \frac{d\sigma_s}{dE'} = \begin{cases} \frac{\sigma_s(E)}{(1-\alpha)E} & \alpha E \leq E' \leq E \\ 0 & otherwise \end{cases}$

More on Moderators

- **Scattering Kernel.**
- **Slowing down density.**
- **Migration length.**
- **Fermi age and continuous fermi model.**

HW 13 (or 6\\)

(Re)-verify

$$\frac{E^{\backslash}}{E}=\frac{A^2+1+2A\cos\theta^{CM}}{(A+1)^2}=\frac{1}{2}\left[(1+\alpha)+(1-\alpha)\cos\theta^{CM}\right]$$

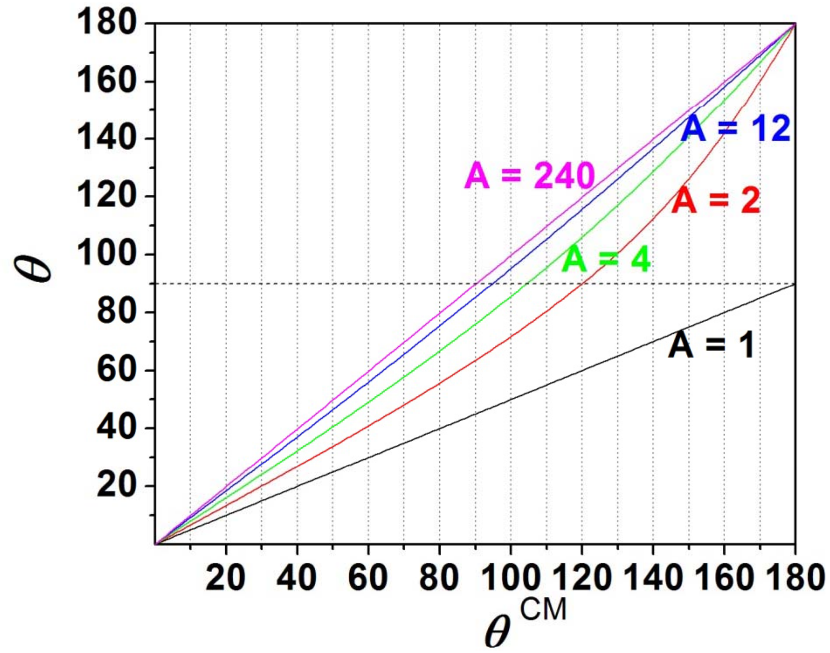
$$=\frac{\left[\cos\theta+\sqrt{A^2-\sin^2\theta}\right]^2}{(A+1)^2}$$

For doing so, you need to verify and use

$$\cos\theta=\frac{1+A\cos\theta^{CM}}{\sqrt{A^2+1+2A\cos\theta^{CM}}}$$

More on Moderators

HW 13 (or 6\) continued…



- Forward scattering is preferred for "practical" moderators (small *A*).
- If isotropic neutron scattering (spherically symmetric) in the **laboratory** frame ▶ average cosine of the scattering angle is zero.

Show that $\overline{\mu} = \overline{\cos(\theta)} = \frac{2}{3A}$

More on Moderators

HW 13 (or 6\\) continued…

Spherically symmetric in CM $\frac{d\sigma_s}{d\Omega^{CM}} = \sigma_s(\theta^{CM}) = \frac{1}{4\pi}\sigma_s(E)$

Show that $\sigma_s(\theta) = \frac{\sigma_s(E)}{4\pi} \frac{(A^{-2} + 2A^{-1}\cos\theta^{CM} + 1)^{3/2}}{1 + A^{-1}\cos\theta^{CM}}$

Try to sketch.

- Neutron **scattering** is isotropic in the laboratory system?! ▶ valid for neutron scattering with heavy nuclei, which is not true for usual thermal reactor moderators (corrections are applied).

Distinguish from

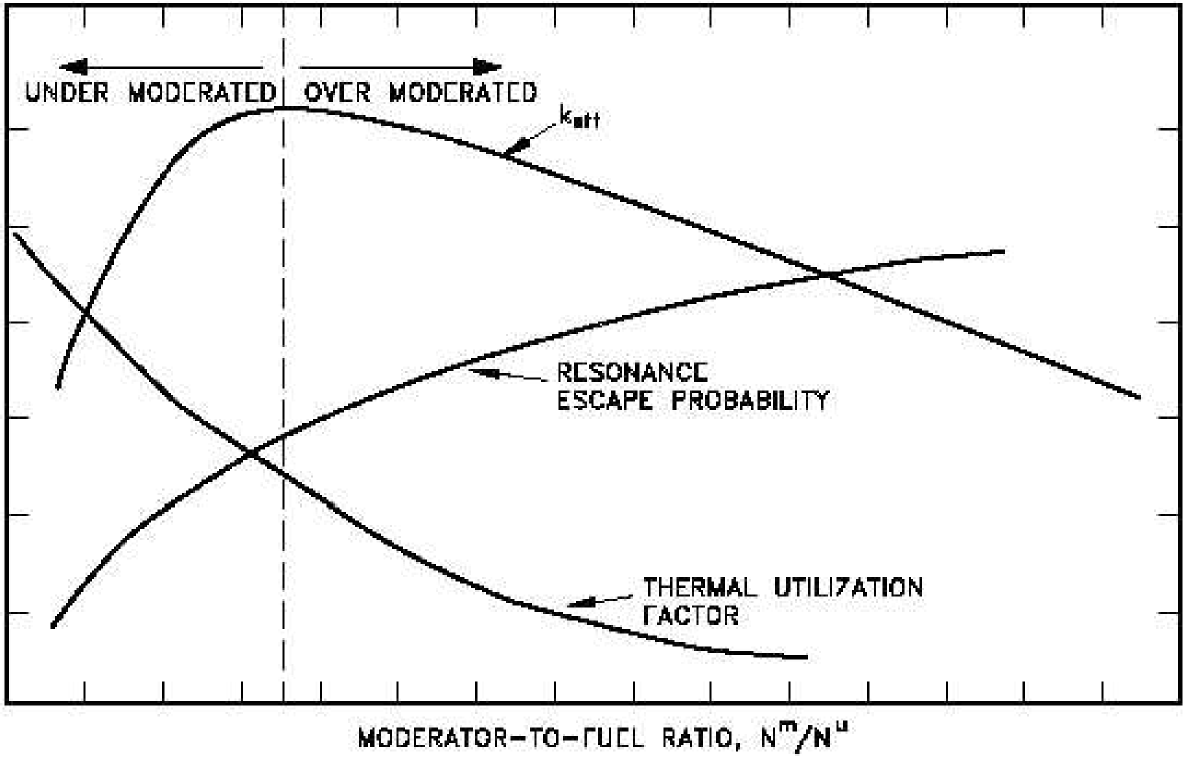
- Angular neutron **distribution.**

More on Moderators

Moderator-to-fuel ratio ≡ N^m/N^u. Self regulation.

- Ratio ↑ p ↑ Σ_a of the moderator ↑ f↓ (leakage ↓).
- Ratio ↓ p ↓ f ↑ (leakage ↑).

- T ↑ ratio ↓ (why).
- Other factors also change.
- Temperature coefficient of reactivity.
- Moderator temperature coefficient of reactivity.



One-Speed Interactions

- Particular ▶ general.

Recall:

- Neutrons don't have a chance to interact with each other (BAU 2007 review test!) ▶ Simultaneous beams, different intensities, same energy:

$$F_t = \Sigma_t (I_A + I_B + I_C + \ldots) = \Sigma_t (n_A + n_B + n_C + \ldots) v$$

- In a reactor, if neutrons are moving in all directions

$$n = n_A + n_B + n_C + \ldots$$

▼

$$R_t = \Sigma_t n v = \Sigma_t \phi$$