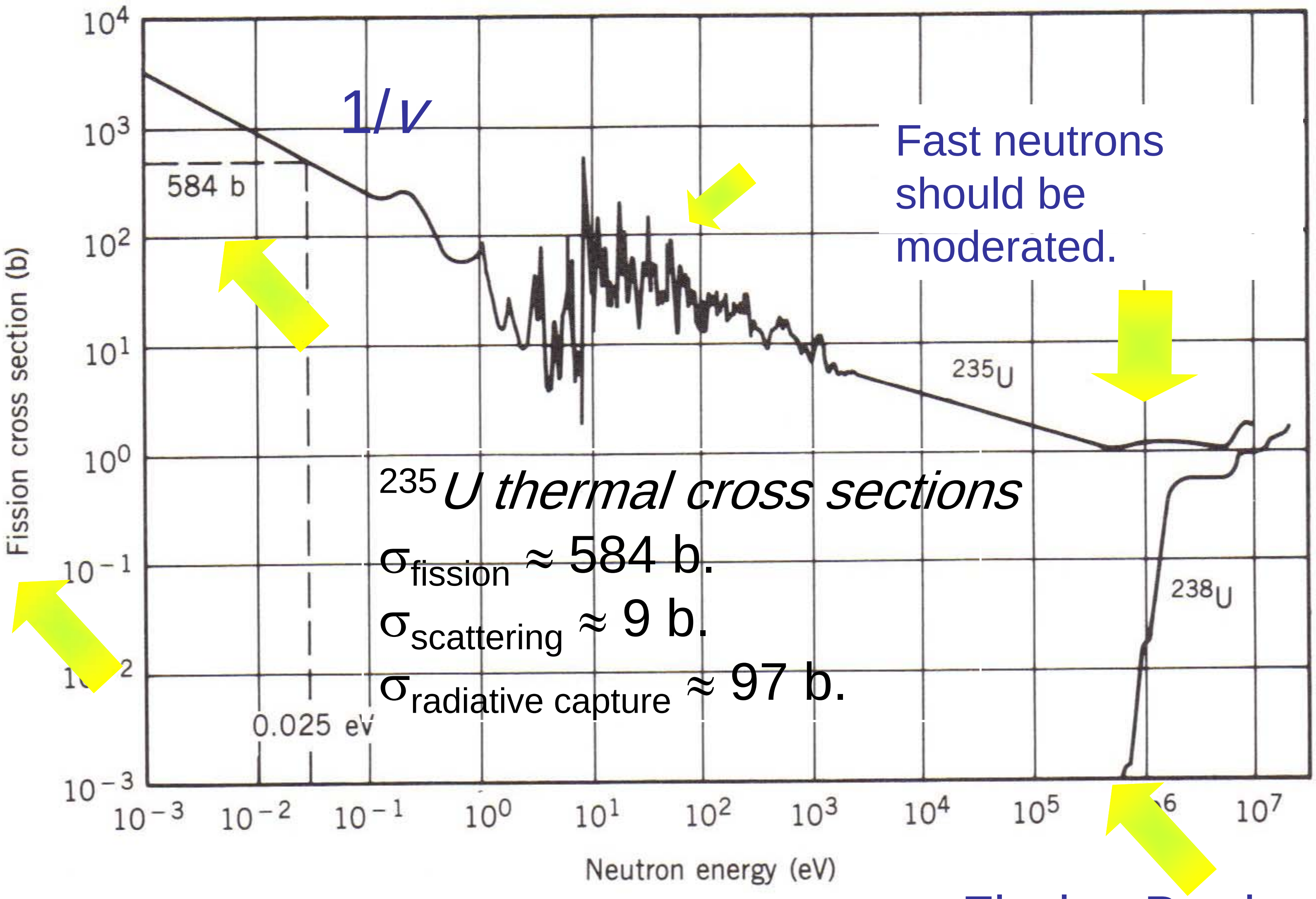
Neutron Cross Section (Different Features)



^{235}U *thermal cross sections*

$\sigma_{fission} \approx 584$ b.

$\sigma_{scattering} \approx 9$ b.

$\sigma_{radiative\ capture} \approx 97$ b.

Fission Barriers

Neutron Induced Reactions

$X(n,b)Y$

$$\sigma_n \propto \lambdabar^2 \left| \langle Y+b | H_{II} | C \rangle \langle C | H_I | X+n \rangle \right|^2$$

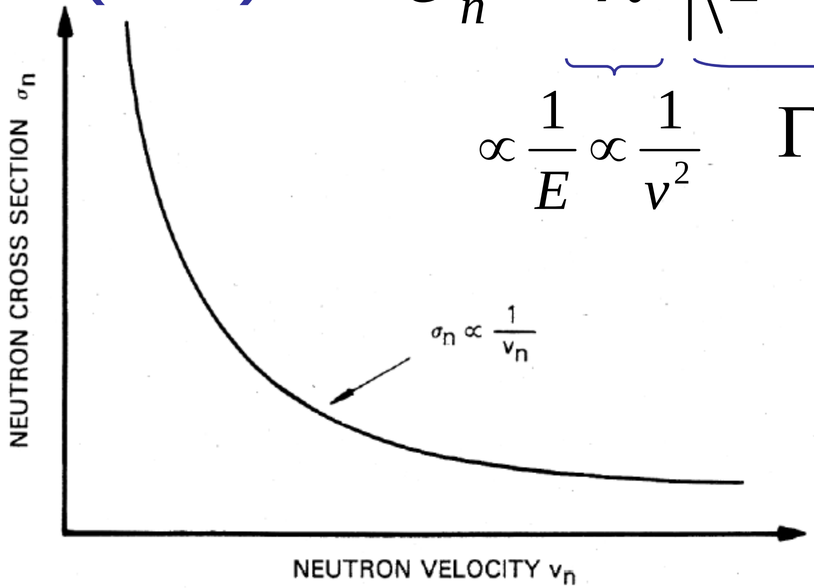
$\lambdabar^2 \propto \frac{1}{E} \propto \frac{1}{v^2}$

$\langle Y+b | H_{II} | C \rangle \rightarrow \Gamma_b(Q+E_n)$

$\langle C | H_I | X+n \rangle \rightarrow \Gamma_n(E_n)$

$$\propto v_n P_{l_n}(E_n)$$

P_{l_n}: Probability to penetrate the potential barrier



For thermal neutrons $Q >> E_n$

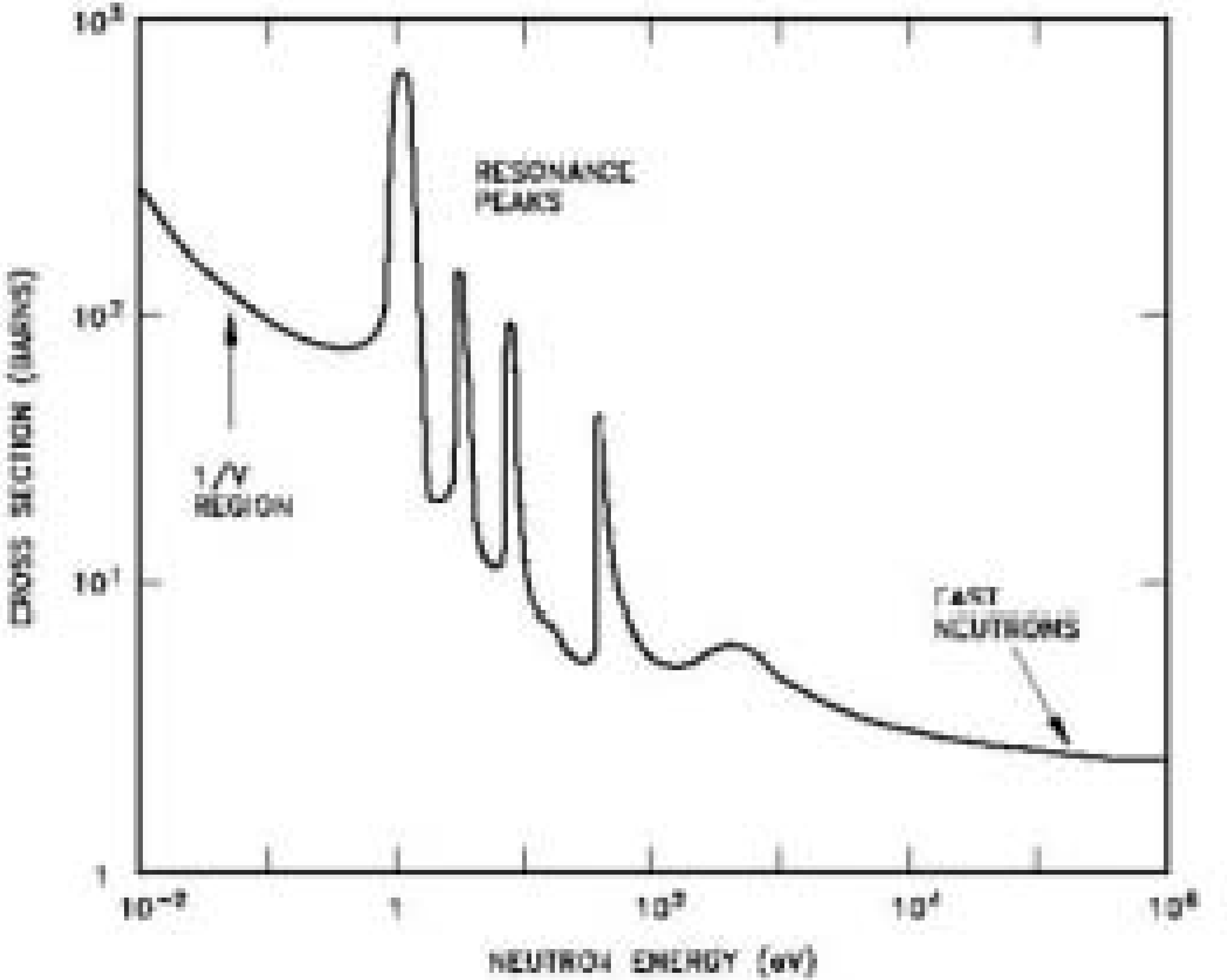
$\Gamma_b(Q) \approx$ constant

$P_o(E_{thermal}) = 1$

$P_{>o}(E_{thermal}) = 0$

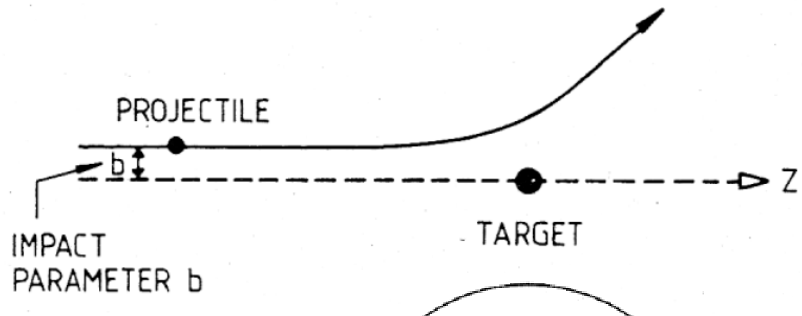
Non-resonant $\quad \sigma_n(E_n) \propto \frac{1}{v}$

Neutron Induced Reactions



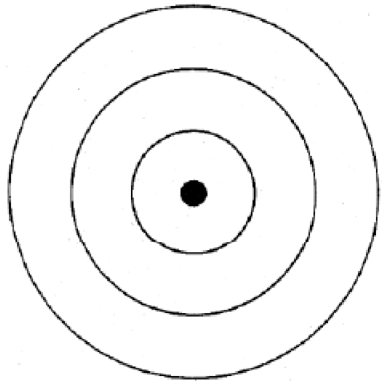
Statistical Factor (Introduction)

$$L = l\hbar = bp = b\frac{\hbar}{\bar{\lambda}}$$



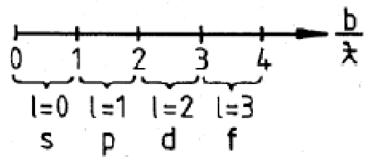
$$b = l\bar{\lambda}$$

$$\sigma_{l,\max} = \pi b_{l+1}^2 - \pi b_l^2 = (2l+1)\pi\bar{\lambda}^2$$



HW 3

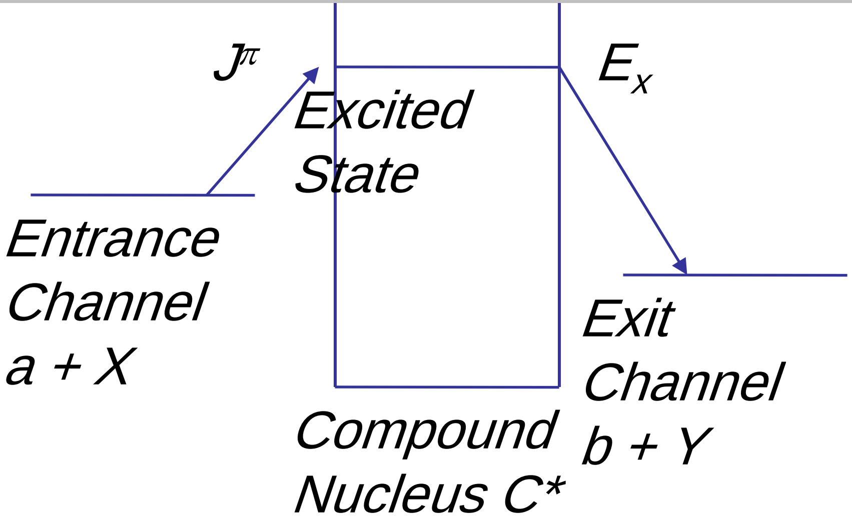
$$\pi\bar{\lambda}^2(b) = \frac{656.7}{\mu(u)E^{CM}(keV)}$$



Generalization

$$\sigma_{\max} = \pi\bar{\lambda}_{aX}^2 \underbrace{\frac{2J+1}{(2J_a+1)(2J_X+1)}(1+\delta_{aX})}_{\omega}$$

Reaction Cross Section



$a + X \rightarrow Y + b \quad Q > 0$

$b + Y \rightarrow X + a \quad Q < 0$

Inverse Reaction

More Generalization

$$\sigma_{aX} = \pi\lambda\!\!\!^{-}{}_{aX}^{2} \frac{2J+1}{(2J_a+1)(2J_X+1)} (1+\delta_{aX}) \left| \langle Y+b | H_{II} | C \rangle \langle C | H_I | a+X \rangle \right|^2$$

QM

Statistical Factor (ω)

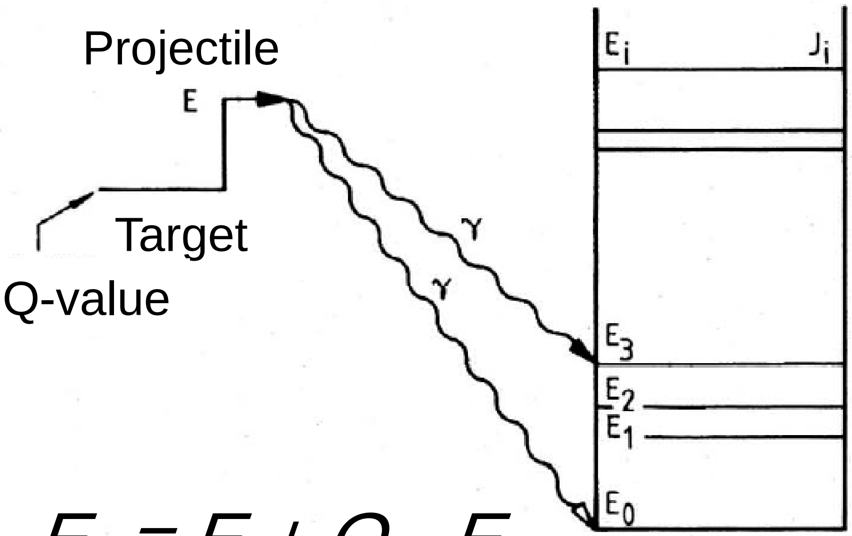
Identical particles

- Nature of force(s).
- Time-reversal invariance.

$$\sigma_{bY} = \pi\lambda\!\!\!^{-}{}_{bY}^{2} \frac{2J+1}{(2J_b+1)(2J_Y+1)} (1+\delta_{bY}) \left| \langle a+X | H_I | C \rangle \langle C | H_{II} | b+Y \rangle \right|^2$$

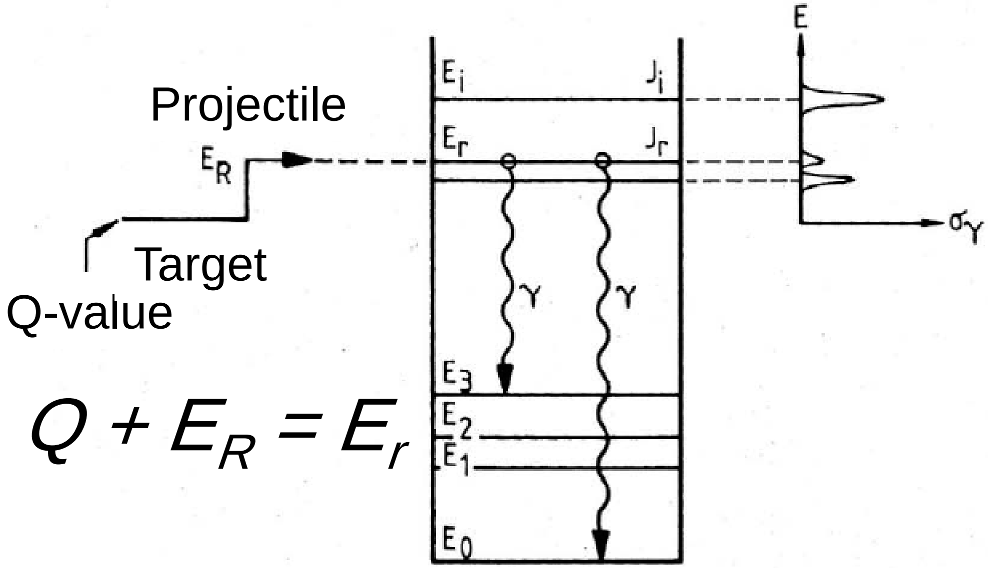
HW 4 $\quad \dfrac{\sigma_{aX}}{\sigma_{bY}} = ??$

Resonance Reactions



$$E_\gamma = E + Q - E_{ex}$$

Direct
Capture
(all energies)

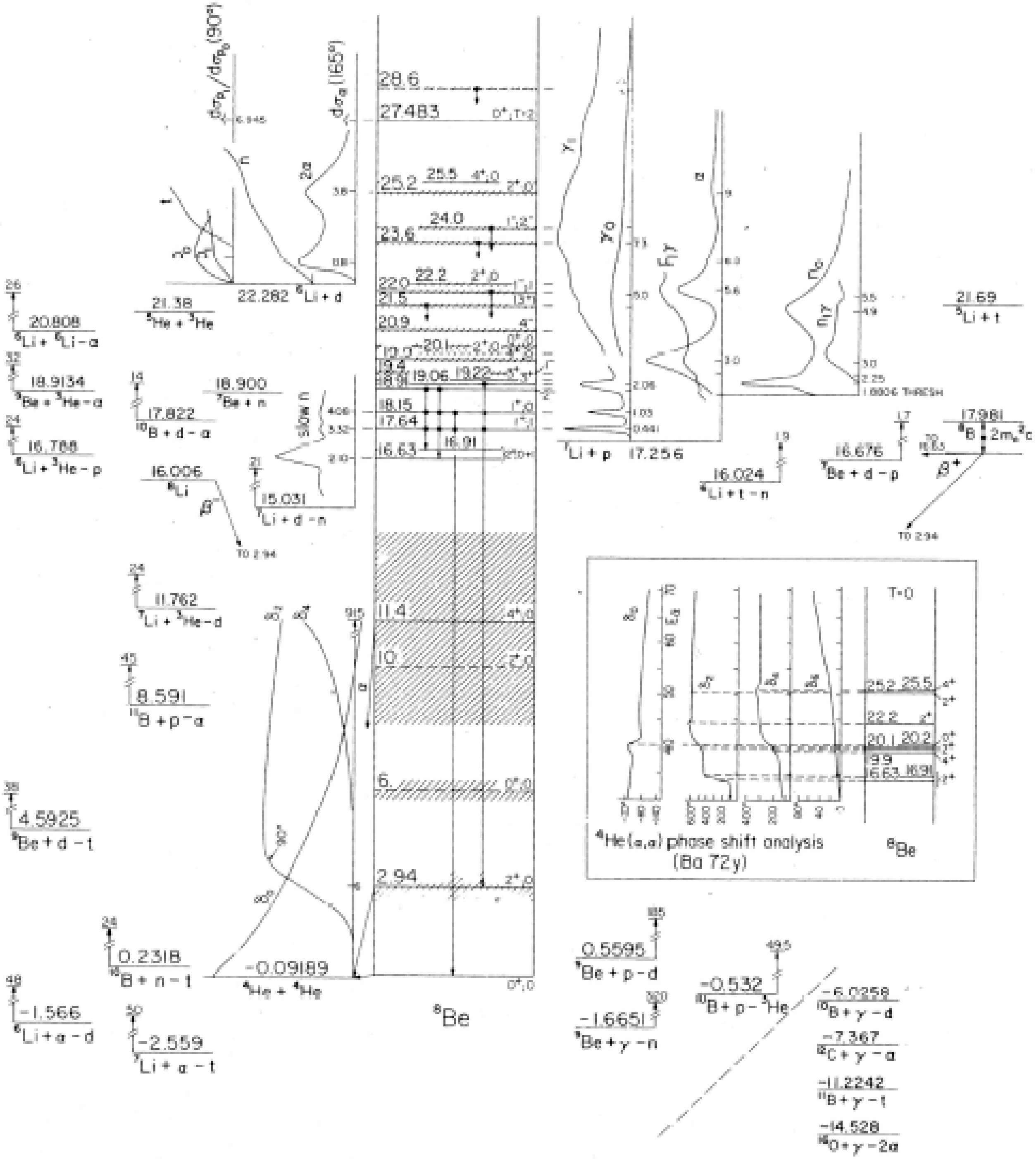


$$Q + E_R = E_r$$

Resonant
Capture
(selected energies
with large X-section)

$$\sigma_\gamma \propto \left| \langle Y | H_\gamma | a + X \rangle \right|^2$$

$$\sigma_\gamma \propto \left| \langle E_f | H_\gamma | E_r \rangle \right|^2 \left| \langle E_r | H_{CN} | a + X \rangle \right|^2$$


28.6
27.483
25.2
25.5
24.0
23.6
22.2
22.0
21.5
20.9
20.1
19.9
19.4
19.06
19.22
18.91
18.15
17.64
16.63
16.91
11.4
10.
6.
2.94
-0.09189
^{4}He + ^{4}He
^{8}Be
22.282 ^{6}Li + d
21.38
^{3}He + ^{5}He
20.808
^{6}Li + ^{6}Li − α
18.9134
^{9}Be + ^{3}He − α
18.900
^{7}Be + n
17.822
^{10}B + d − α
16.788
^{6}Li + ^{3}He − p
16.006
^{8}Li
β$^-$
TO 2.94
15.031
^{7}Li + d − n
11.762
^{7}Li + ^{3}He − d
8.591
^{11}B + p − α
4.5925
^{9}Be + d − t
0.2318
^{10}B + n − t
-1.566
^{6}Li + α − d
-2.559
^{7}Li + α − t
^{7}Li + p
17.256
21.69
^{5}Li + t
17.981
^{8}B
2m$_e$c^2
β$^+$
TO 2.94
16.676
^{9}Be + d − p
16.024
^{6}Li + t − n
0.5595
^{9}Be + p − d
-1.6651
^{9}Be + γ − n
-0.532
^{10}B + p − ^{3}He
-6.0258
^{10}B + γ − d
-7.367
^{12}C + γ − α
-11.2242
^{11}B + γ − t
-14.528
^{16}O + γ − 2α
slow n
1.8806 THRESH
T=0
^{4}He(α,α) phase shift analysis
(Ba 72y)
^{8}Be

Resonance Reactions

Damped Oscillator

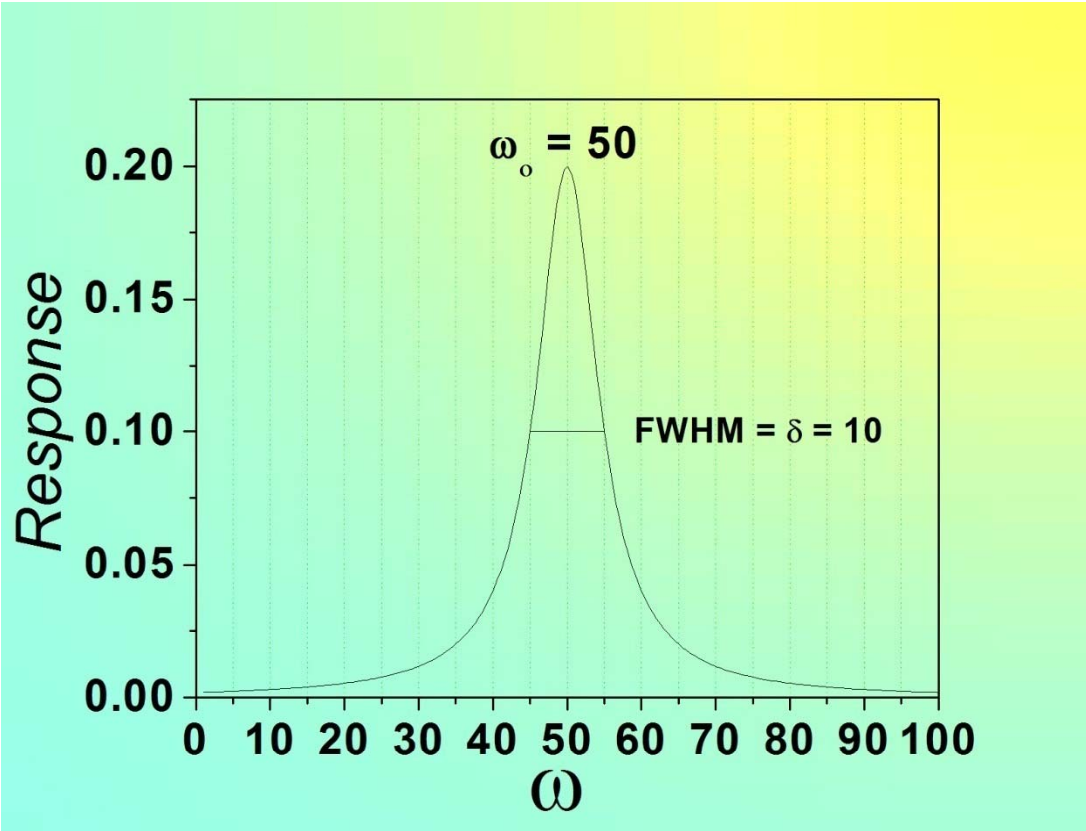
Oscillator strength

$$response \propto \frac{f}{(\omega - \omega_o)^2 + (\frac{\delta}{2})^2}$$

$$\delta = \frac{1}{t_0}$$

Damping factor

eigenfrequency



$$\sigma(E) \propto \frac{\Gamma_a \Gamma_b}{(E - E_R)^2 + (\frac{\Gamma}{2})^2}$$

$$\Gamma t_o = \hbar$$

Resonance Reactions

$$\sigma(E) = \pi \lambda\!\!\!^{-}{}_{aX}{}^{2} \frac{2J+1}{(2J_a+1)(2J_X+1)} (1+\delta_{aX}) \frac{\Gamma_a \Gamma_b}{(E-E_R)^2 + (\frac{\Gamma}{2})^2}$$

Breit-Wigner formula

$$\Gamma = \Gamma_a + \Gamma_b$$

- All quantities in CM system
- Only for isolated resonances.

$\sigma_R \propto \Gamma_a \Gamma_b$ ← Reaction

$\sigma_e \propto \Gamma_a \Gamma_a$ ← Elastic scattering

Usually $\Gamma_a >> \Gamma_b$.

$$\frac{\sigma_R}{\sigma_e} = \frac{\Gamma_b}{\Gamma_a}$$

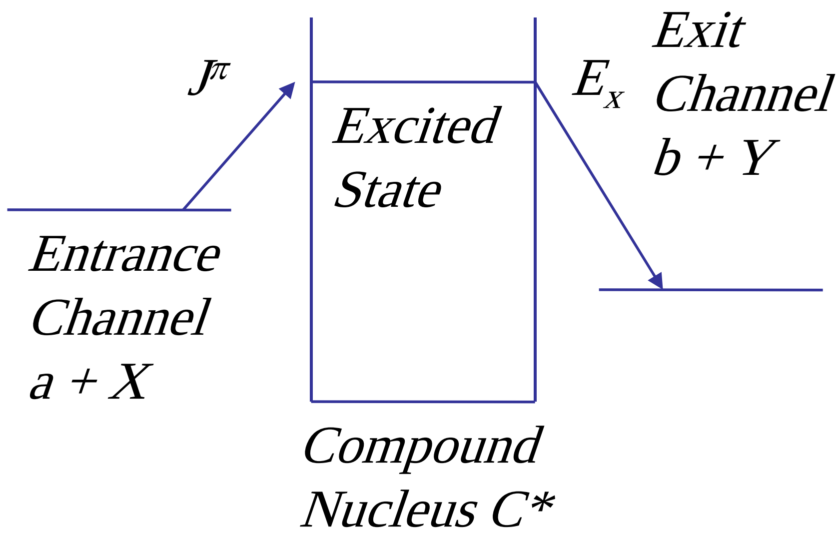
HW 5 When does σ_R take its maximum value?

Resonance Reactions

$$J_a + J_X + l = J$$

$$(-1)^l\, \pi(J_a)\, \pi(J_X) = \pi(J)$$

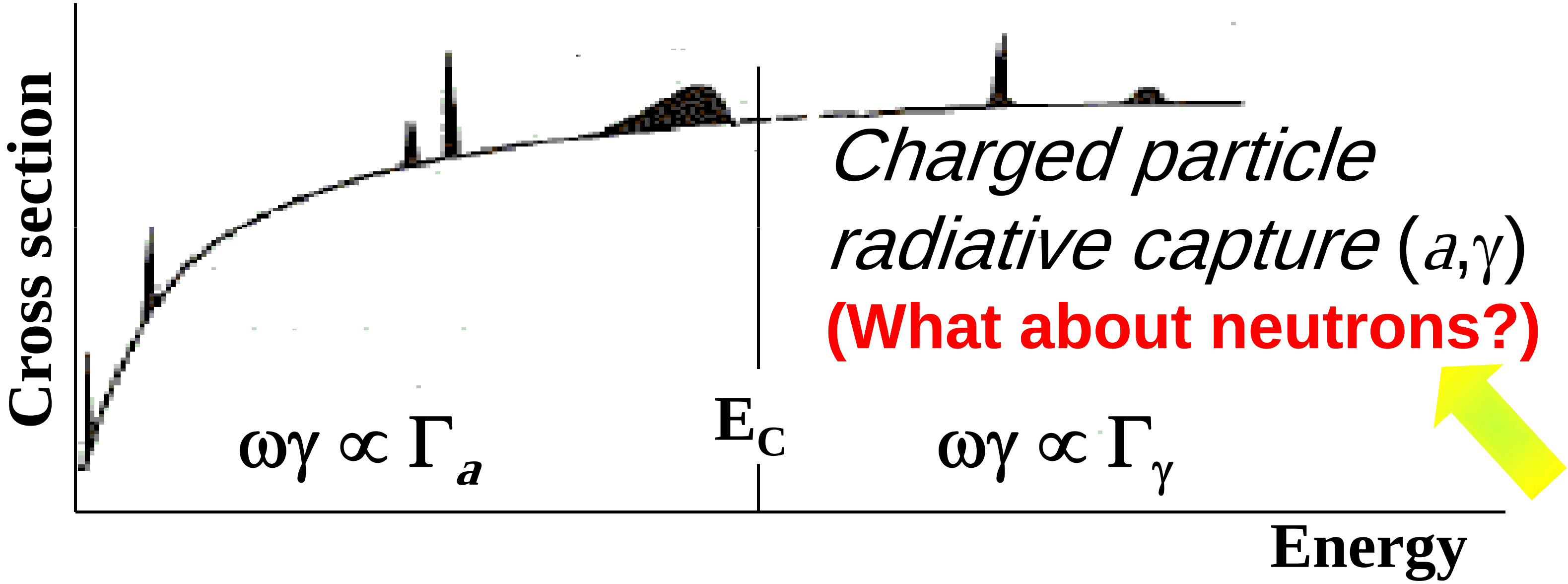
$$(-1)^l = \pi(J)$$ Natural parity.



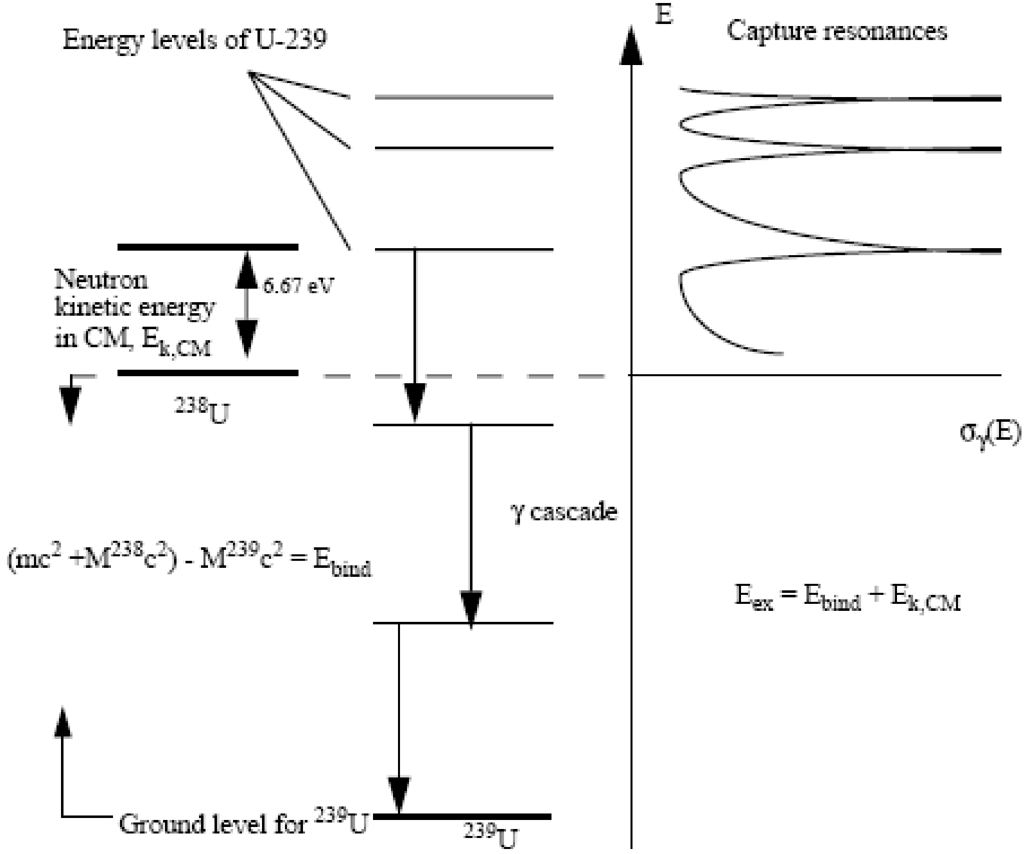
Resonance Reactions

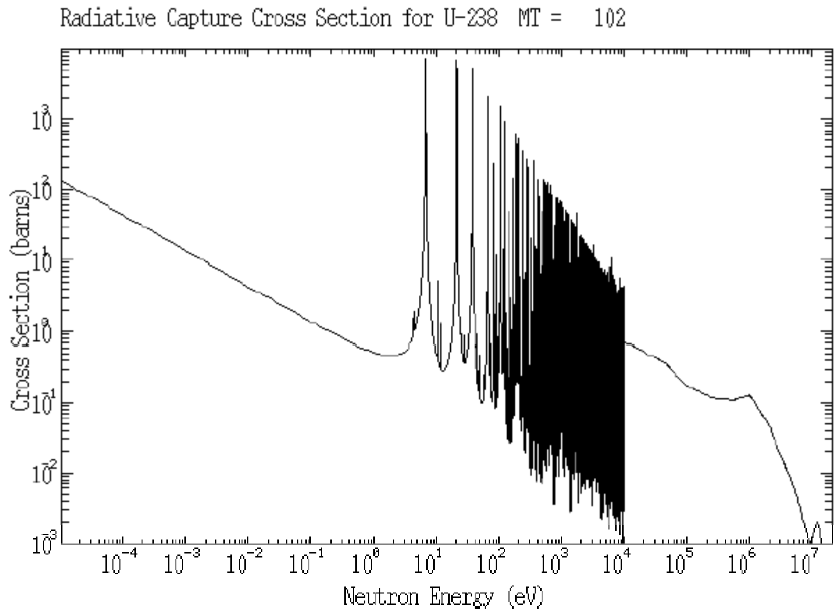
What is the "Resonance Strength" …?
What is its significance?
In what units is it measured?

$$\omega\gamma = \frac{2J+1}{(2J_a+1)(2J_X+1)}(1+\delta_{aX})\frac{\Gamma_a\Gamma_b}{\Gamma}$$



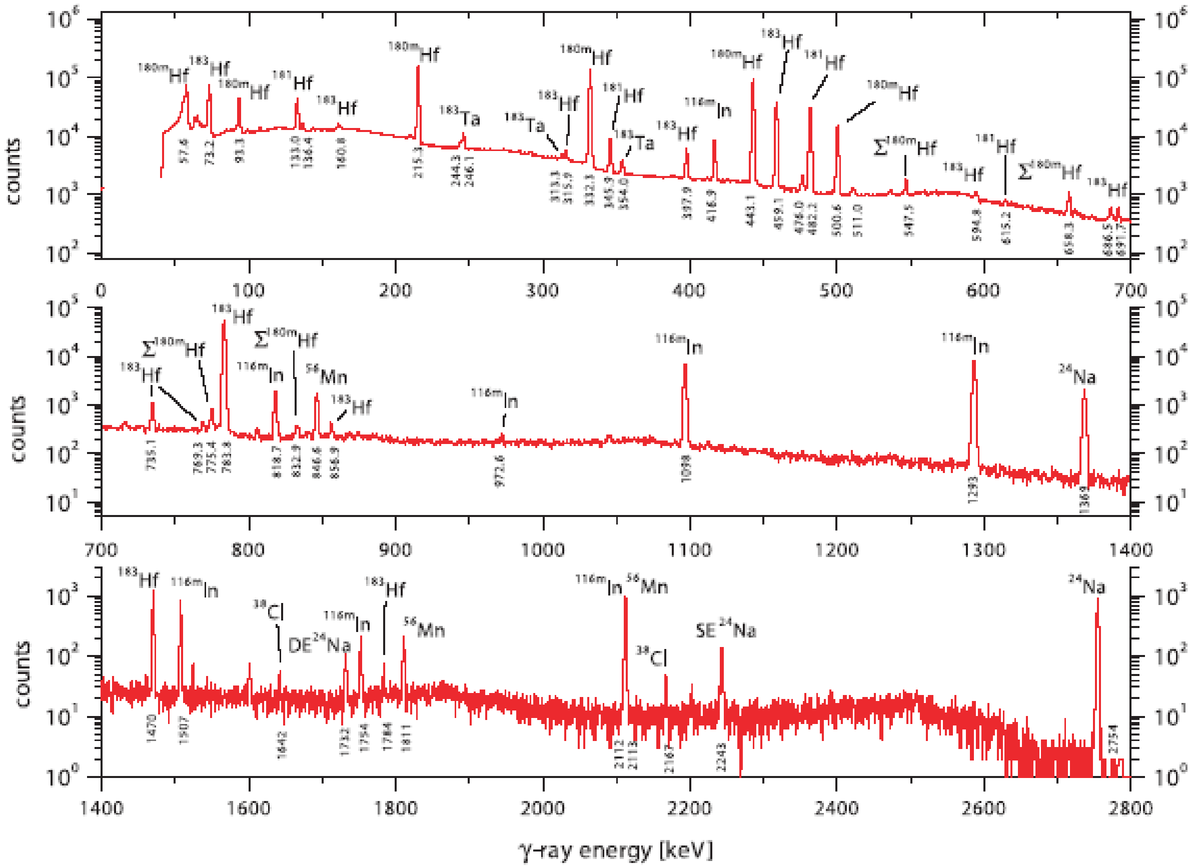
Neutron Resonance Reactions

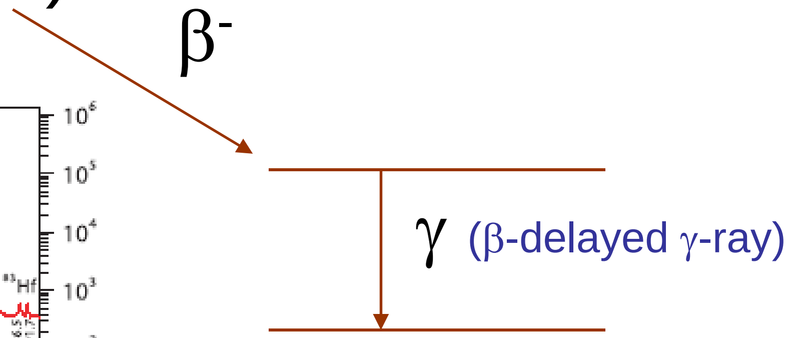




Neutron Activation Analysis

$$(Z,A) + n \rightarrow (Z, A+1)$$





$$(Z+1, A+1)$$

Project 1

NAA and U

Neutron Flux and Reaction Rate

Recall $F_t = n\, v\, \sigma_t\, N = I\, \Sigma_t$

Simultaneous beams, different intensities, **same energy**.

$$F_t = \Sigma_t (I_A + I_B + I_C + \ldots) = \Sigma_t (n_A + n_B + n_C + \ldots) v$$

In a **reactor**, if neutrons are moving in **all directions**

$$n = n_A + n_B + n_C + \ldots$$

$$F_t = \Sigma_t n v$$

Locally

neutron flux $\phi = nv$

Not talking about a beam anymore.

Reaction Rate $R_t \equiv F_t = \Sigma_t \phi = \phi / \lambda_t \; (= nvN\sigma_t)$

same energy

Neutron Flux and Reaction Rate

Different energies

Density of neutrons with energy between E and $E+dE$

Units! $n(E)dE$

Reaction rate for those "monoenergetic" neutrons

$$dR_t = \Sigma_t(E)\, n(E)dE\, v(E)$$

$$n = \int_0^\infty n(E)dE$$

$$\phi = \int_0^\infty \underbrace{\phi(E)}dE = \int_0^\infty \underbrace{n(E)\upsilon(E)}dE$$

$$R_t = \int_0^\infty \Sigma_t(E)n(E)\upsilon(E)dE = \int_0^\infty \Sigma_t(E)\phi(E)dE$$

$$R_i = \int_0^\infty \Sigma_i(E)\phi(E)dE$$

Neutron Flux and Reaction Rate

In general, neutron flux depends on:

- **Neutron energy, *E*.**
- **Neutron spatial position, *r*.**
- **Neutron angular direction, Ω.**
- **Time, *t*.**

Various kinds of neutron fluxes (depending on the degree of detail needed).

Time-dependent and time-independent angular neutron flux.

$$\phi(r, E, \Omega, t)$$

$$\phi(r, E, \Omega)$$