Fuel Depletion

$$N \sim 10^{22}\, cm^{-3},\ \phi \sim 10^{14}\, cm^{-2} s^{-1}$$

Time scale: Days and months.

- More depletion ▶ increase steady state flux by means of reducing absorbers.
- For a given fuel isotope $\dfrac{\partial N_f(\vec{r},t)}{\partial t} = -N_f(\vec{r},t)\sigma_a^f \phi(\vec{r},t)$
- For **constant flux ϕ_0** the solution is

$$N_f(\vec{r},t) = N_f(\vec{r},0)e^{-\sigma_a^f \phi_0(\vec{r})t} = N_f(\vec{r},0)e^{-\sigma_a^f \Phi(\vec{r},t)}$$

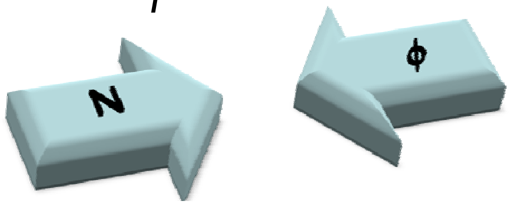
Exponential burnup

Neutron fluence

- For **time varying flux**

$$N_f(\vec{r},t) = N_f(\vec{r},0)e^{-\sigma_a^f \int_0^t \phi(\vec{r},t')dt'} = N_f(\vec{r},0)e^{-\sigma_a^f \Phi(\vec{r},t)}$$

Solve numerically.



Fuel Depletion

- **Constant power.**

$$P(\vec{r},t) = w\overset{\downarrow}{N_f}(\vec{r},t)\sigma_f^f\overset{\uparrow}{\phi}(\vec{r},t) = P(\vec{r},0) = P_0(\vec{r})$$

w: Energy released per fission

$N_f(\vec{r},t)\sigma_f^f\phi(\vec{r},t)$: Fission rate

$$N_f(\vec{r},t)\phi(\vec{r},t) = N_f(\vec{r},0)\phi(\vec{r},0)$$

$$\Sigma_f(\vec{r},t)\phi(\vec{r},t) = \Sigma_f(\vec{r},0)\phi(\vec{r},0)$$

- Power ~ flux only over short time periods during which N_f is constant.

$$\frac{\partial N_f(\vec{r},t)}{\partial t} = -\underset{\downarrow}{N_f}(\vec{r},t)\sigma_a^f\underset{\uparrow}{\phi}(\vec{r},t) \approx -\frac{P_0(\vec{r})}{w}$$

Linear depletion!

$$\sigma_f^f \approx \sigma_a^f$$

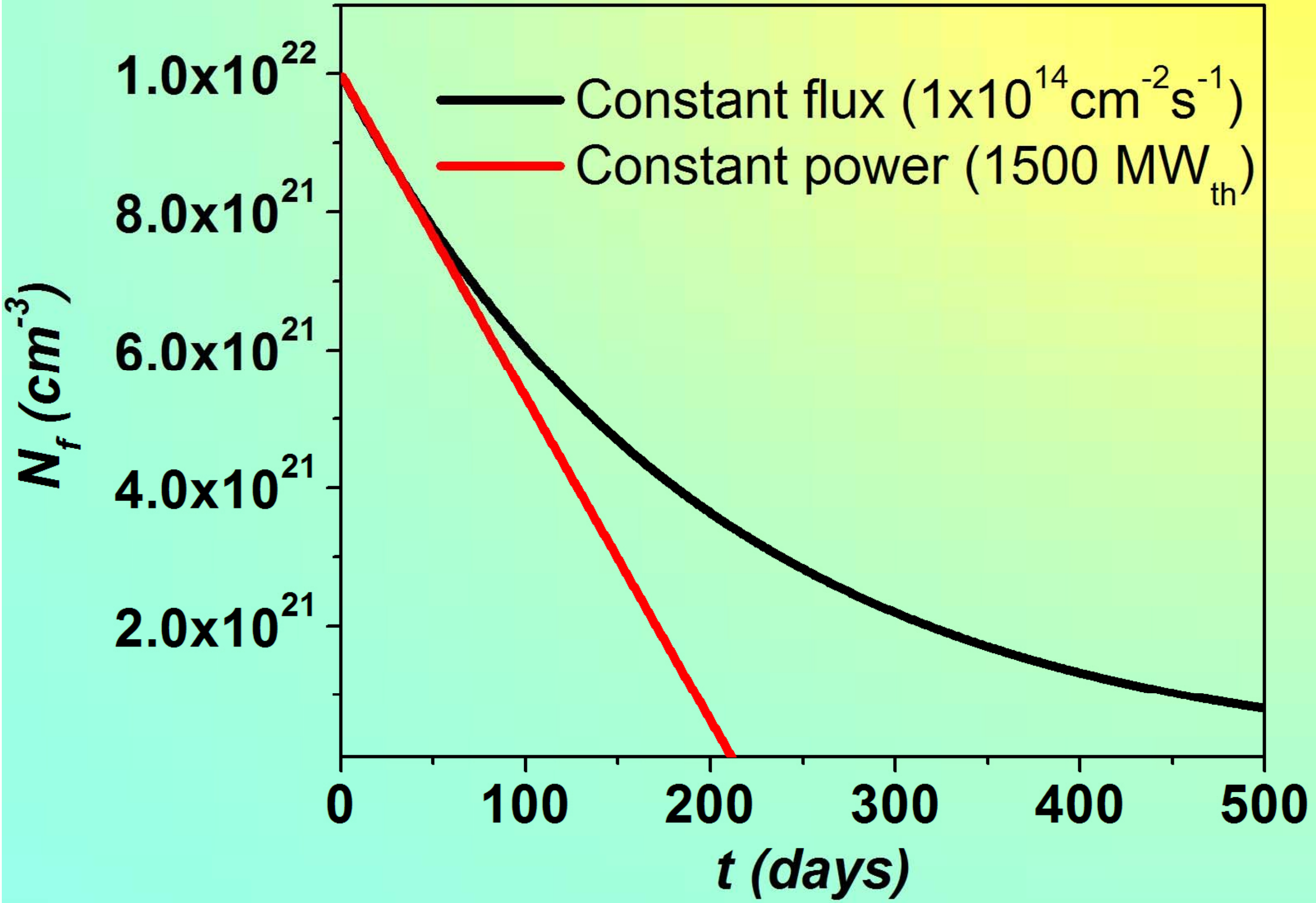
- The solution is obviously

$$N_f(\vec{r},t) \approx N_f(\vec{r},0) - \frac{P_0(\vec{r})}{w}t$$

Fuel Depletion

<u>HW 31</u>

Do the calculations for different flux and power levels.



Poisoning and Fuel Depletion

Infinite, critical homogeneous reactor.

$$k_\infty = \varepsilon \eta f \rho = \varepsilon \eta \rho \frac{\overset{\downarrow}{\Sigma_a^f(t)}}{\underset{\downarrow}{\Sigma_a^f(t)} + \Sigma_a^{clad} + \Sigma_a^{moderator}(t) + \underset{\uparrow}{\Sigma_a^{poison}(t)} + \underset{\text{thus}\downarrow}{\Sigma_a^{control}(t)}}$$

Constant power $N_f(\vec{r},t) \approx N_f(\vec{r},0) - \frac{P_0(\vec{r})}{w} t$

$$= N_f(\vec{r},0) - N_f(\vec{r},t)\sigma_a^f \phi(\vec{r},t)t$$

$$= N_f(\vec{r},0) - N_f(\vec{r},0)\sigma_a^f \phi(\vec{r},0)t$$

$$= N_f(\vec{r},0)\left[1 - \sigma_a^f \phi(\vec{r},0)t\right]$$

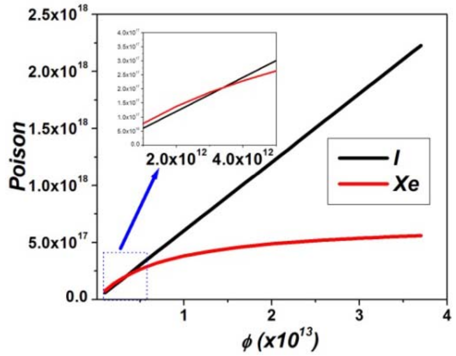
$$\phi(\vec{r},t) = \frac{N_f(\vec{r},0)}{N_f(\vec{r},t)}\phi(\vec{r},0) = \frac{\phi(\vec{r},0)}{1-\sigma_a^f \phi(\vec{r},0)t}$$

$$\Sigma_a^f(\vec{r},t) = \Sigma_a^f(\vec{r},0)\left[1 - \sigma_a^f \phi(\vec{r},0)t\right]$$

Poisoning and Fuel Depletion

$$Xe(t) = \overbrace{\frac{(\gamma_I + \gamma_{Xe})\Sigma_f \phi_0}{\lambda_{Xe} + \sigma_a^{Xe}\phi_0}}^{Xe_\infty}(1 - e^{-(\lambda_{Xe} + \sigma_a^{Xe}\phi_0)t}) + \frac{\gamma_I \Sigma_f \phi_0}{\lambda_{Xe} - \lambda_I + \sigma_a^{Xe}\phi_0}(e^{-(\lambda_{Xe} + \sigma_a^{Xe}\phi_0)t} - e^{-\lambda_I t})$$

$\Sigma_f \phi_0$ → Constant



Constant

$$\Sigma_a^{Xe}(\vec{r},t) = \sigma_a^{Xe} Xe_\infty(\vec{r},t) = \frac{\overbrace{(\gamma_I + \gamma_{Xe})\Sigma_f(\vec{r},0)\phi(\vec{r},0)}^{\text{Constant}}}{\frac{\lambda_{Xe}}{\sigma_a^{Xe}} + \phi(\vec{r},t)}$$

$$\Sigma_a^{Sm}(\vec{r},t) \approx \sigma_a^{Sm} \gamma_{Sm} \Sigma_f(\vec{r},0)\phi(\vec{r},0)t$$

$$\phi(\vec{r},t) = \frac{\phi(\vec{r},0)}{1 - \sigma_a^f \phi(\vec{r},0)t}$$

- Other fission products (poisons) with less capture cross sections.

Poisoning and Fuel Depletion

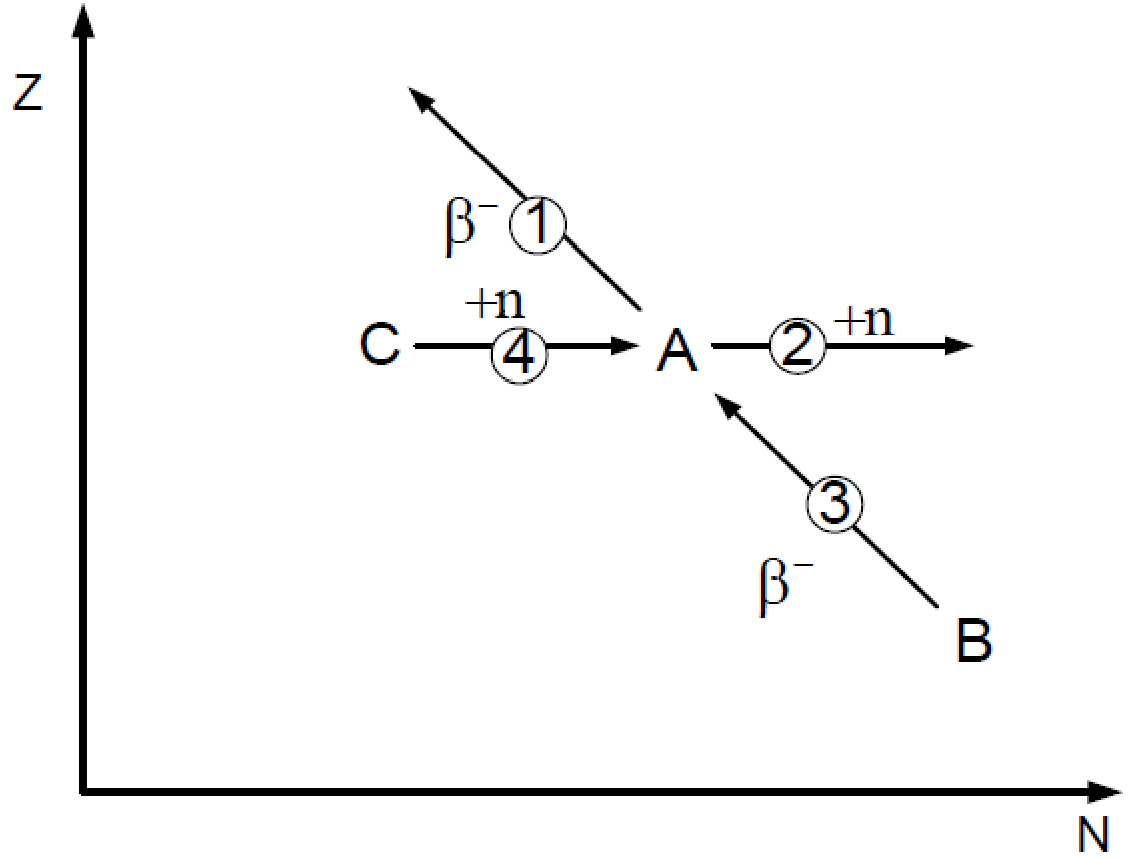
- Now we know all macroscopic cross sections.

$$k_\infty = \varepsilon\eta f\rho = \varepsilon\eta\rho \frac{\overset{\downarrow}{\Sigma_a^f(t)}}{\underset{\downarrow}{\Sigma_a^f(t)} + \Sigma_a^{clad} + \Sigma_a^{\text{mod}erator}(t) + \underset{\uparrow}{\Sigma_a^{poison}(t)} + \Sigma_a^{control}(t)}$$

- When there are no absorbers left to remove, we need to refuel.
- Absorbers are not only control rods.
- All fuel nuclei should be considered.
- For each species, all sources and sinks should be taken into account.
- Online loading ▶ environmental.
- ^3H.

Until = 0. Solve for t to get upper limit for "core loading lifetime". Damaged fuel…!

Poisoning and Fuel Depletion



$$\frac{dN_A}{dt} = -\lambda_A N_A - \phi\sigma_A N_A + \lambda_B N_B + \phi\sigma_\gamma^C N_C + F(t)$$

Fuel loading

Poisoning and Fuel Depletion

- **Some poisons are intentionally introduced into the reactor.**
  - Fixed burnable poisons.

    B, Gd.

    More uniform distribution than rods, more intentionally localized than shim.

    Flatter flux.
  - Soluble poisons (chemical shim) with caution.

    Boric acid (soluble boron, solbor) in coolant.

    Boration and dilution.

    Scram emergency shutdown (sodium polyborate or gadolinium nitrate).

    Power shaping.
  - Non-burnable poisons.

    Chain of absorbers or self shielding.

Delayed Precursors

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$

$$- \Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

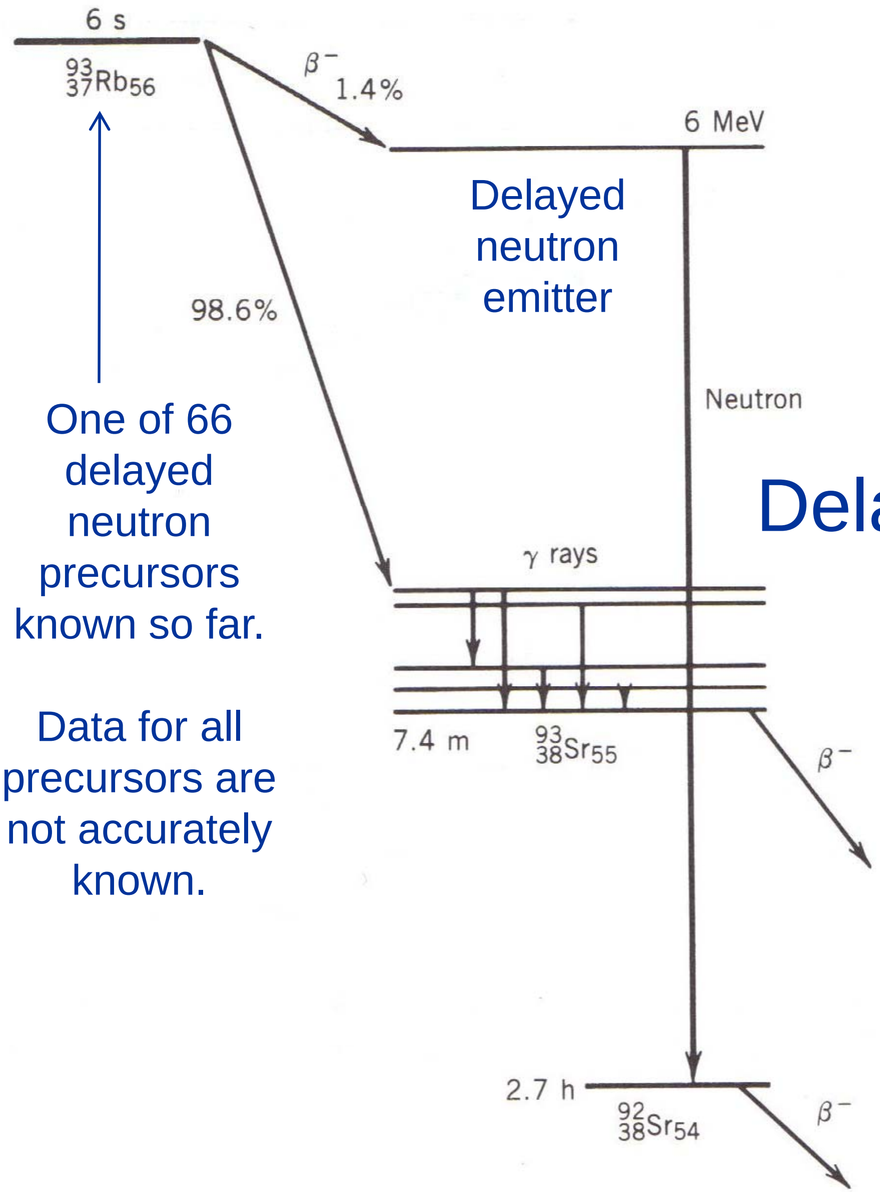
- For one-group

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = \nu \Sigma_f(\vec{r})\phi(\vec{r},t) + S^{ext}$$

$$- \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla} \bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

- **What about delayed neutrons?**

Delayed Precursors

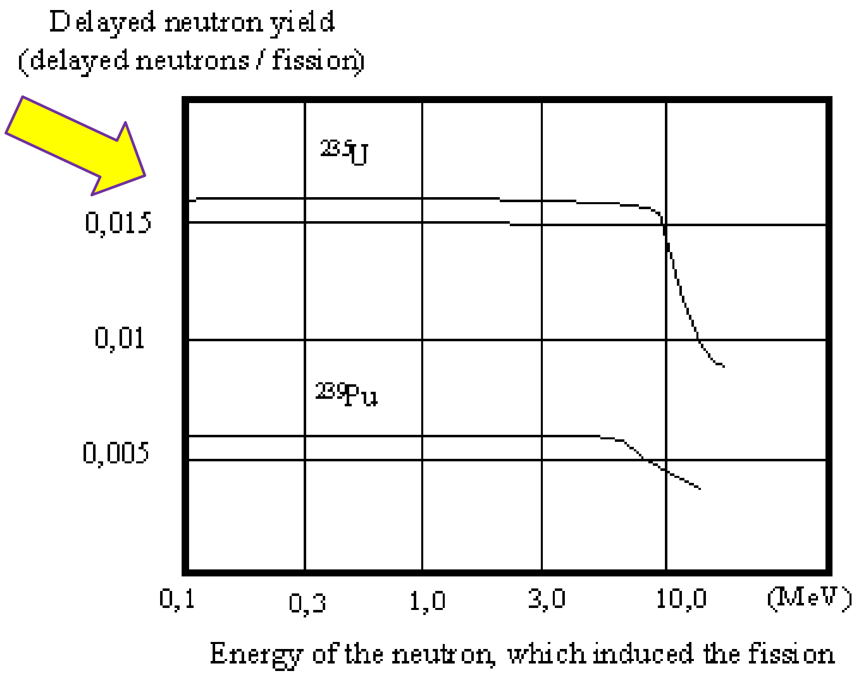


One of 66 delayed neutron precursors known so far.

Data for all precursors are not accurately known.

$$\nu = \nu_p + \nu_d$$

Delayed neutron fraction $\beta = \frac{\nu_d}{\nu}$



Delayed Precursors

Fissile nucleus	Delayed neutron / 100 fissions
^{233}U	0.667
^{235}U	1.621
^{238}U*	4.39
^{239}Pu	0.628
^{240}Pu*	0.95
^{241}Pu	1.52
^{242}Pu*	2.21

Increases with *N*.

Data for thermal neutron induced fission, except for * fast neutron induced fission.

Delayed Precursors

Group # →	1	2	3	4	5	6
$T_{1/2}$ (s)	54.51	21.84	6.0	2.23	0.496	0.179
λ_I	0.0127	0.031	0.1155	0.310	1.397	3.871
β_i/β	.038	0.213	0.188	0.407	0.128	0.026
β_I	0.0002641	0.00148035	0.0013066	0.00282865	0.0008896	0.0001807

^{235}U $\qquad \beta < 0.7\% \cong 0.016 / \nu$

ν_p

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = \underbrace{(1-\beta)\nu}_{\nu_p}\Sigma_f(\vec{r})\phi(\vec{r},t) + \sum_{i=1}^{6}\lambda_i C_i + S^{ext} - \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla}\bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

$$\frac{\partial C_i(\vec{r},t)}{\partial t} = -\lambda_i C_i(\vec{r},t) + \beta_i \nu \Sigma_f(\vec{r})\phi(\vec{r},t)$$

Delayed Precursors

- The multi-group equation now becomes

Different energy spectra

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g^p(1-\beta)\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \chi_g^C\sum_{i=1}^{6}\lambda_i C_i(\vec{r},t)$$

$$+\sum_{g'=1}^{G}\Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$

$$-\Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

$$\frac{\partial C_i(\vec{r},t)}{\partial t} = -\lambda_i C_i(\vec{r},t) + \beta_i\sum_{g'=1}^{G}\nu_{g'}\Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t)$$

Full Blown Diffusion Equation

- In **steady state**

$$\lambda_i C_i(\vec{r},t) = \beta_i \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t)$$

$$0 = \chi_g^p (1-\beta) \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r}) + \chi_g^C \beta \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r})$$

$$+ \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r}) + S_g^{ext} - \Sigma_{ag}(\vec{r})\phi_g(\vec{r}) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r}) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r})$$

Significance of χ_g^C depends on whether we have fine or course energy groups.

$$0 = \left[\chi_g^p + (\chi_g^C - \chi_g^p)\beta\right] \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r})$$

$$+ \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r}) + S_g^{ext} - \Sigma_{ag}(\vec{r})\phi_g(\vec{r})$$

$$- \Sigma_{sg}(\vec{r})\phi_g(\vec{r}) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r})$$