

# More on One-Speed Diffusion

## Spherical Bare Reactor (one-speed diffusion)

$$\text{Cube} \quad \frac{6a^2}{a^3} > \frac{4\pi a^2}{\frac{4}{3}\pi a^3} \quad \text{Sphere}$$

Minimum leakage ► minimum fuel to achieve criticality.

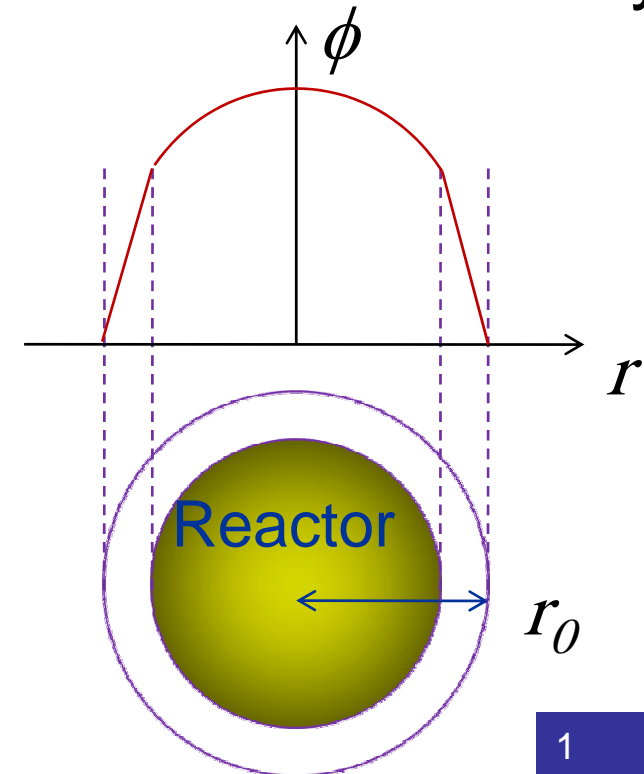
**HW 22**  $\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} + B^2\phi = 0$

$$\phi = \frac{A}{r} \cos Br + \frac{C}{r} \sin Br$$

▼

$$\phi = \frac{C}{r} \sin \frac{\pi r}{r_0}, \quad r_0 = \frac{\pi}{B}$$

Continue!

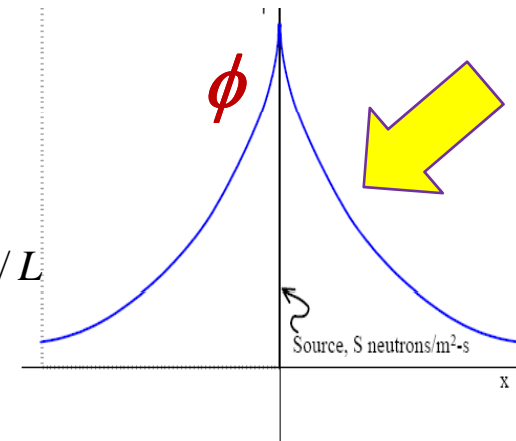


# More on One-Speed Diffusion

## HW 23

Infinite planer source in an infinite medium.

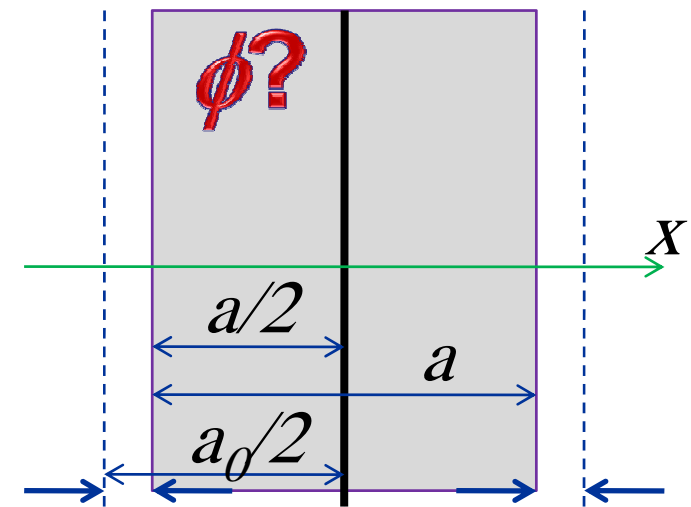
$$\frac{d^2 \phi(x)}{dx^2} - \frac{1}{L^2} \phi = -\frac{S \delta(x)}{D} \quad \blacktriangleright \quad \phi(x) = \frac{SL}{2D} e^{-|x|/L}$$



## HW 24

Infinite planer source in a finite medium.

$$\phi = \frac{SL}{2D} \frac{\sinh[(a_0 - 2|x|)/2L]}{\cosh(a_0/2L)}$$



# More on One-Speed Diffusion

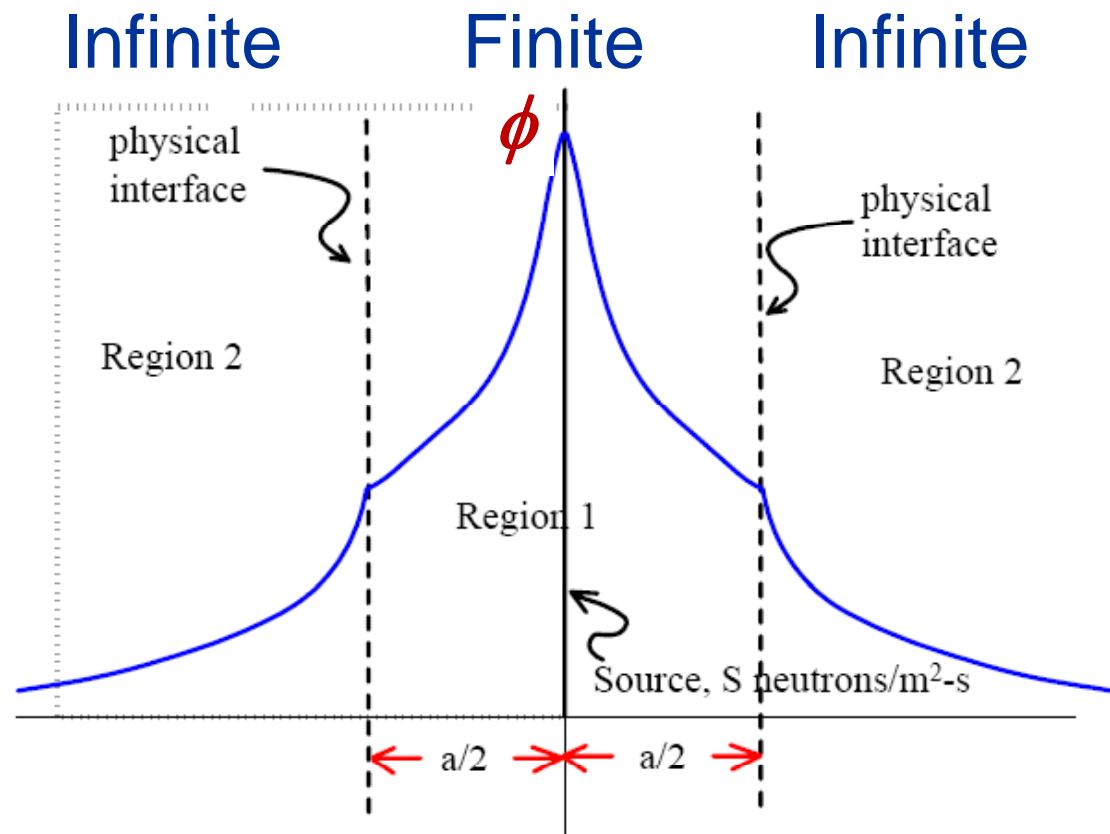
## Infinite planer source in a multi-region medium.

$$\phi_1(\pm a/2) = \phi_2(\pm a/2)$$

$$D_1 \left. \frac{d\phi_1}{dx} \right|_{x=\pm a/2} = D_2 \left. \frac{d\phi_2}{dx} \right|_{x=\pm a/2}$$

+ more BC

## Project 2



# Back to Multiplication Factor

Recall:

Things to be used later...!

$$k_{\infty} = fp\varepsilon\eta, \quad \frac{k_{eff}}{k_{\infty}} = P_{non-leak} \quad \blacktriangleright \quad k_{eff} = fp\varepsilon\eta P_{non-leak}$$

- Fast from thermal,  $\eta = \frac{1}{\sum_a} \sum_i \nu(i) \Sigma_f(i)$
- Fast from fast,  $\varepsilon$ .
- Thermal from fast,  $p$ .

- Thermal available for fission  $f = \frac{\sum_a^{fuel}}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator} + \sum_a^{poison}}$

Recall:

## Thinking QUIZ

- For each thermal neutron absorbed, how many fast neutrons are produced?

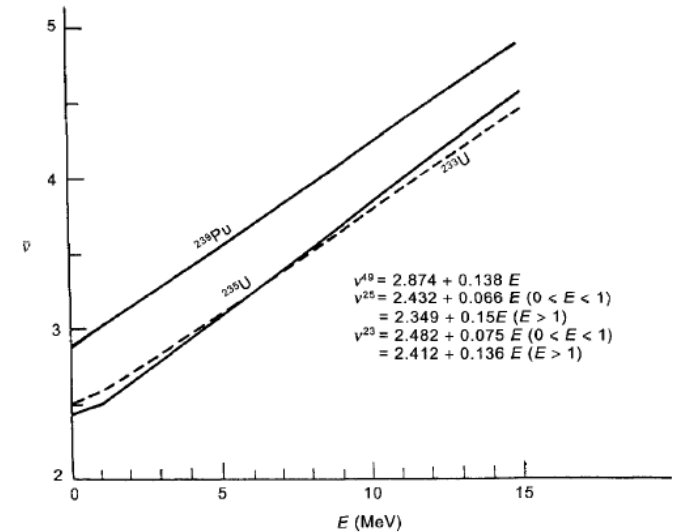
# Two-Group Neutron Diffusion

- **Introductory to multi-group (Hence crude).**
- All neutrons are either in a fast or in a thermal energy group.
- Boundary between two groups is set to  $\sim 1$  eV.
- **Thermal neutrons** diffuse in a medium and cause fission, are captured, or leak out from the system.
- Source for thermal neutrons is provided by the slowing down of fast neutrons (born in fission).
- **Fast neutrons** are lost by slowing down due to elastic scattering in the medium, or leak out from the system, or due to fission or capture.
- Source for fast neutrons is thermal and fast neutron fission.

# Two-Group Neutron Diffusion

$$\phi_1(\vec{r}) = \int_{1\text{eV}}^{10\text{MeV}} \phi(E, \vec{r}) dE \quad \textit{Fast}$$

$$\phi_2(\vec{r}) = \int_0^{1\text{eV}} \phi(E, \vec{r}) dE \quad \textit{Thermal}$$



$$k_{eff} = \frac{\nu_1 \sum_{f1} \phi_1 + \nu_2 \sum_{f2} \phi_2}{-D_1 \nabla^2 \phi_1 - D_2 \nabla^2 \phi_2 + \sum_{a1} \phi_1 + \sum_{a2} \phi_2}$$

# Two-Group Neutron Diffusion

Fast

$$0 = S_1(\vec{r}) - \sum_{a1} \phi_1(\vec{r}) + D_1 \nabla^2 \phi_1(\vec{r})$$

Depends on  
thermal and fast  
fluxes.

**Removal** cross section  
= fission + capture +  
scattering to group 2

Fast diffusion  
coefficient

$$0 = \nu \sum_{f1} \phi_1(\vec{r}) + \nu \sum_{f2} \phi_2(\vec{r}) - \sum_{a1} \phi_1(\vec{r}) + D_1 \nabla^2 \phi_1(\vec{r})$$

or

$$0 = \frac{k_\infty}{\rho} \sum_{a2} \phi_2(\vec{r}) - \sum_{a1} \phi_1(\vec{r}) + D_1 \nabla^2 \phi_1(\vec{r})$$

# Two-Group Neutron Diffusion

Thermal

$$0 = S_2(\vec{r}) - \sum_{a_2} \phi_2(\vec{r}) + D_2 \nabla^2 \phi_2(\vec{r})$$

Depends on fast flux.

Thermal absorption cross section = fission + capture.

Thermal diffusion coefficient

$$0 = \sum_{s1 \rightarrow 2} \phi_1(\vec{r}) - \sum_{a_2} \phi_2(\vec{r}) + D_2 \nabla^2 \phi_2(\vec{r})$$

or

$$0 = \rho \sum_{a_1} \phi_1(\vec{r}) - \sum_{a_2} \phi_2(\vec{r}) + D_2 \nabla^2 \phi_2(\vec{r})$$



# Two-Group Neutron Diffusion

$$0 = \frac{k_{\infty}}{\rho} \sum_{a_2} \phi_2(\vec{r}) - \sum_{a_1} \phi_1(\vec{r}) + D_1 \nabla^2 \phi_1(\vec{r})$$

$$0 = \rho \sum_{a_1} \phi_1(\vec{r}) - \sum_{a_2} \phi_2(\vec{r}) + D_2 \nabla^2 \phi_2(\vec{r})$$

- A coupled system of equations; both depend on both fluxes.
- Recall also, for a steady state system:

$$\nabla^2 \phi_1(\vec{r}) + B^2 \phi_1(\vec{r}) = 0$$

$$\nabla^2 \phi_2(\vec{r}) + B^2 \phi_2(\vec{r}) = 0$$

# Two-Group Neutron Diffusion

Homogeneous system ► Determinant of coefficients matrix = 0

$$\begin{vmatrix} -\Sigma_{a1} - D_1 B^2 & \frac{k_\infty}{\rho} \Sigma_{a2} \\ \rho \Sigma_{a1} & -\Sigma_{a2} - D_2 B^2 \end{vmatrix} = 0$$

Review Cramer's rule!  
Do we need it here?

$$(-\Sigma_{a1} - D_1 B^2)(-\Sigma_{a2} - D_2 B^2) - \frac{k_\infty}{\rho} \Sigma_{a2} \rho \Sigma_{a1} = 0$$

$$(\Sigma_{a1} + D_1 B^2)(\Sigma_{a2} + D_2 B^2) - k_\infty \Sigma_{a2} \Sigma_{a1} = 0$$

$$\left(\frac{1}{L_{Fast}^2} + B^2\right)\left(\frac{1}{L_{Thermal}^2} + B^2\right) - k_\infty \frac{1}{L_{Fast}^2} \frac{1}{L_{Thermal}^2} = 0$$

$$(1 + B^2 L_{Fast}^2)(1 + B^2 L_{Thermal}^2) - k_\infty = 0$$

# Two-Group Neutron Diffusion

$$(1 + B^2 L_{Fast}^2)(1 + B^2 L_{Thermal}^2) - k_{\infty} = 0$$

$$\frac{k_{\infty}}{(1 + B^2 L_{Fast}^2)(1 + B^2 L_{Thermal}^2)} = 1$$

$k_{eff}$  for a critical reactor

$$\frac{k_{eff}}{k_{\infty}} = P_{non-leak}^{Fast} P_{non-leak}^{Thermal} = \frac{1}{B^2 L_{Thermal}^2 + 1} \frac{1}{B^2 L_{Fast}^2 + 1}$$

For large reactors

$$\frac{k_{\infty}}{1 + B^2 (L_{Fast}^2 + L_{Thermal}^2)} = 1 \Rightarrow B^2 = \frac{k_{\infty} - 1}{M^2}$$

Migration Length

# Two-Group Neutron Diffusion

$$M^2 = L_{Thermal}^2 + L_{Fast}^2$$

If any ↑  
leakage  
↑.

$$L_{Thermal}^2 = \frac{D}{\Sigma_a} = \frac{\lambda_{tr}}{3\Sigma_a} = \frac{1}{3\Sigma_a \Sigma_{tr}}$$

$$\text{Fermi age} \equiv L_{Fast}^2 = \frac{n}{3\Sigma_s \Sigma_{tr}}$$

- Slowing down density.
- Fermi model.

**Do it.**

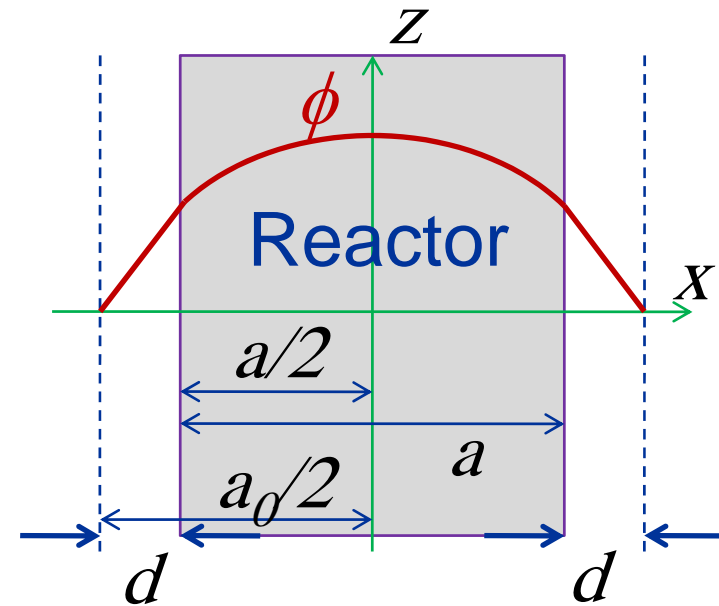
# Reactor Model: One-Group

- Before considering multi-group.
- So far we did 1-D.
- Back to **one-group** but extend to **3-D**.

## HW 21!

For the homogeneous infinite slab reactor, extend the criticality condition that you found in HW 21.

$$B_g^2 = \left( \frac{\pi}{a_0} \right)^2 = B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{\nu \Sigma_f - \Sigma_a}{D}$$



**1-D**

# Reactor Model: One-Group

- In 3-D

$$\frac{d^2 \phi(x)}{dx^2} + B^2 \phi(x) = 0 \quad \blacktriangleright \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} + B^2 \phi = 0$$

$\frac{\nu \Sigma_f - \Sigma_a}{D}$

$$\phi = \phi_0 \cos Bx \quad \blacktriangleright \quad \phi = \phi_0 \cos B_x x \cos B_y y \cos B_z z$$

$$B_g^2 = \left( \frac{\pi}{a_0} \right)^2 = B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{\nu \Sigma_f - \Sigma_a}{D} \quad \blacktriangleright$$

$$B_g^2 = B_x^2 + B_y^2 + B_z^2 = \underbrace{\left( \frac{\pi}{a_0} \right)^2 + \left( \frac{\pi}{b_0} \right)^2 + \left( \frac{\pi}{c_0} \right)^2}_{B_m^2} = B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{\nu \Sigma_f - \Sigma_a}{D}$$

Critical dimensions (size), for the given material properties, predicted by the model.

# Reactor Model: One-Group

- Transient case.

$$\frac{1}{\nu} \frac{\partial}{\partial t} \phi(\vec{r}, t) = S(\vec{r}, t) - \sum_a(\vec{r}) \phi(\vec{r}, t) + \vec{\nabla} \cdot D(\vec{r}) \vec{\nabla} \phi(\vec{r}, t)$$

$$t! \quad \sum_a^{fuel} = \sum_f^{fuel} + \sum_\gamma^{fuel}$$

Moderator, structure,  
coolant, fuel, ...

- **Delayed neutrons!!**
- **Reflectors!!**
- For **homogeneous 1-D**:

$$\frac{1}{\nu} \frac{\partial}{\partial t} \phi(x, t) = S(x, t) - \sum_a \phi(x, t) + D \frac{\partial^2}{\partial x^2} \phi(x, t)$$

$$\nu \sum_f \phi(x, t)$$

# Reactor Model: One-Group

$$\frac{1}{v} \frac{\partial}{\partial t} \phi(x, t) = \nu \Sigma_f \phi(x, t) - \Sigma_a \phi(x, t) + D \frac{\partial^2}{\partial x^2} \phi(x, t)$$

## HW 25

Separation of variables:  $\phi(x, t) = \psi(x)T(t)$

$$\frac{1}{v} \psi \frac{\partial T}{\partial t} = \nu \Sigma_f \psi T - \Sigma_a \psi T + DT \frac{\partial^2 \psi}{\partial x^2}$$

$$\frac{1}{T} \frac{\partial T}{\partial t} = \frac{\nu}{\psi} \left[ D \frac{\partial^2 \psi}{\partial x^2} + (\nu \Sigma_f - \Sigma_a) \psi \right] \equiv -\lambda = \text{constant}$$

= 0 for steady state.

**Show that**  $T(t) = T(0)e^{-\lambda t}$ ,  $\lambda = \nu(\Sigma_a + DB^2 - \nu \Sigma_f)$



# Reactor Model: One-Group

## HW 25 (continued)

$$\phi\left(\pm \frac{a_0}{2}\right) = 0 \quad \text{try} \quad \psi_n(x) = \cos B_n x \quad B_n^2 = \left(\frac{n\pi}{a_0}\right)^2$$

eigenvalues  $\lambda_n = \nu(\Sigma_a + DB_n^2 - \nu \Sigma_f)$

Solution  $\phi(x, t) = \sum_{n \text{ odd}} A_n e^{-\lambda_n t} \cos\left(\frac{n\pi x}{a_0}\right)$

?

Initial condition  $\phi(x, 0) = \sum_{n \text{ odd}} A_n \cos\left(\frac{n\pi x}{a_0}\right)$

Show that  $A_n = \frac{2}{a_0} \int_{-\frac{a_0}{2}}^{+\frac{a_0}{2}} \phi(x, 0) \cos\left(\frac{n\pi x}{a_0}\right) dx$

# Reactor Model: One-Group

$$B_n^2 = \left( \frac{n\pi}{a_0} \right)^2 \quad \blacktriangleright \quad B_1^2 < B_3^2 < B_5^2 < \dots$$

$$\lambda_n = \nu(\Sigma_a + DB_n^2 - \nu \Sigma_f) \quad \blacktriangleright \quad \lambda_1^2 < \lambda_3^2 < \lambda_5^2 < \dots$$

$$\lambda_1 = \nu(\Sigma_a + DB_1^2 - \nu \Sigma_f) \quad \text{Slowest decaying eigenvalue.}$$



$$\phi(x, t) \cong A_1 e^{-\lambda_1 t} \cos\left(\frac{\pi x}{a_0}\right) = A_1 e^{-\lambda_1 t} \cos B_1 x$$

# Reactor Model: One-Group

For steady state  $\lambda_1 = \nu(\Sigma_a + DB_1^2 - \nu \Sigma_f) = 0$

**Criticality**  $B_1^2 = B_g^2 = \frac{\nu \Sigma_f - \Sigma_a}{D} \equiv B_m^2 \quad \lambda_1 = 0$

**Super criticality**  $B_g^2 < B_m^2 \quad LE \downarrow \quad \lambda_1 < 0$

**Sub criticality**  $B_g^2 > B_m^2 \quad LE \uparrow \quad \lambda_1 > 0$