One-Speed Interactions

- Particular ▶ general.

Recall:

- Neutrons don't have a chance to interact with each other (2007 review test!) ▶ Simultaneous beams, different intensities, same energy:

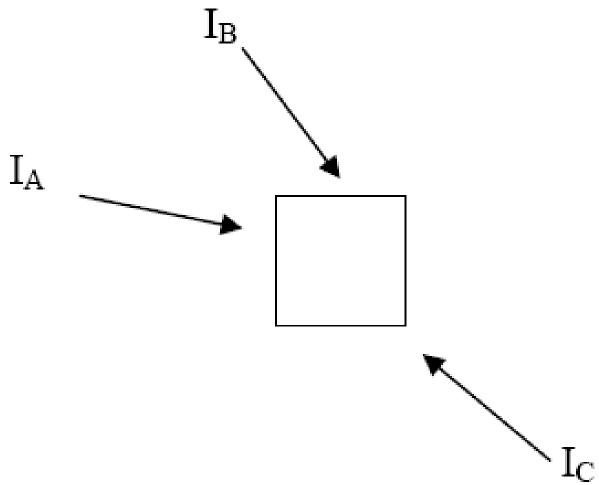
$$F_t = \Sigma_t (I_A + I_B + I_C + \ldots) = \Sigma_t (n_A + n_B + n_C + \ldots) v$$

- In a reactor, if neutrons are moving in all directions

$$n = n_A + n_B + n_C + \ldots$$

▼

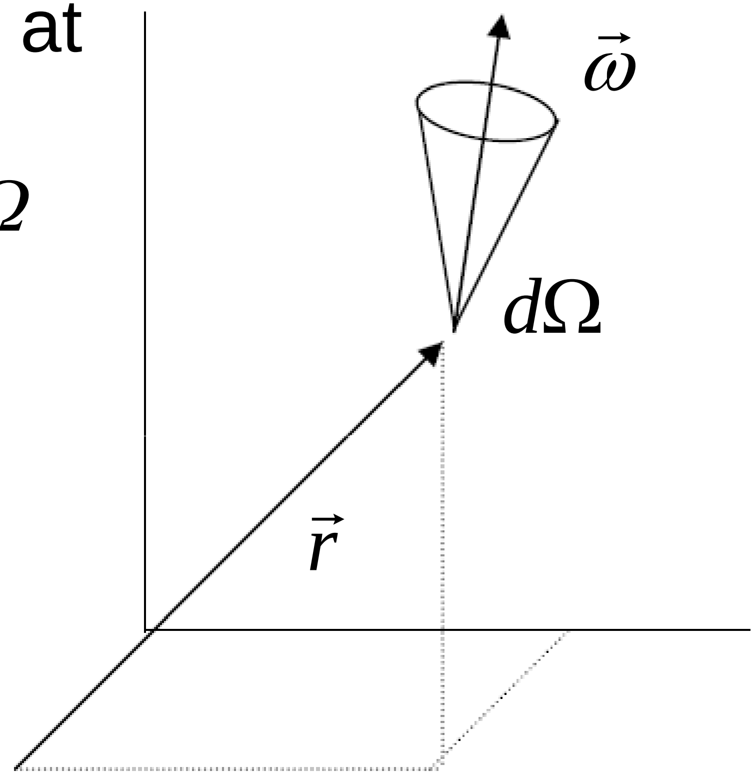
$$R_t = \Sigma_t n v = \Sigma_t \phi$$



One-Speed Interactions

$n(\vec{r},\vec{\omega})d\Omega \equiv$ Neutrons per cm^3 at $\boldsymbol{r}$ whose velocity vector lies within $d\Omega$ about ω.

$$n(\vec{r}) = \int_{4\pi} n(\vec{r},\vec{\omega})d\Omega$$



- Same argument as before ▶

$$dI(\vec{r},\vec{\omega}) = n(\vec{r},\vec{\omega})v d\Omega$$

$$dF(\vec{r},\vec{\omega}) = \Sigma_t(\vec{r})dI(\vec{r},\vec{\omega})$$

Scalar

$$R(\vec{r}) = F(\vec{r}) = \int_{\omega} dF(\vec{r},\vec{\omega}) = \Sigma_t(\vec{r})v\int_{4\pi} n(\vec{r},\vec{\omega})d\Omega = \Sigma_t(\vec{r})vn(\vec{r}) = \Sigma_t(\vec{r})\phi(\vec{r})$$

where $\phi(\vec{r}) = v\int_{4\pi} n(\vec{r},\vec{\omega})d\Omega$

Multiple Energy Interactions

- Generalize to include energy

$n(\vec{r},E,\vec{\omega})dEd\Omega$ ≡ Neutrons per cm^3 at $\boldsymbol{r}$ with energy interval (E, $E+dE$) whose velocity vector lies within $d\Omega$ about ω.

$$n(\vec{r},E)dE = \int_{4\pi} n(\vec{r},E,\vec{\omega})d\Omega dE \qquad n(\vec{r}) = \int_0^\infty \int_{4\pi} n(\vec{r},E,\vec{\omega})d\Omega dE$$

$$R_t(\vec{r},E)dE = \Sigma_t(\vec{r},E)n(\vec{r},E)v(E)dE = \Sigma_t(\vec{r},E)\phi(\vec{r},E)dE$$

$$R_t(\vec{r}) = \int_0^\infty \Sigma_t(\vec{r},E)\phi(\vec{r},E)dE$$

Scalar

Thus knowing the material properties Σ_t and the neutron flux ϕ, *both* as functions of space and energy, we can calculate the interaction rate throughout the reactor.

Neutron Current

- Similarly $R_S(\vec{r}) = \int_0^{\infty} \Sigma_S(\vec{r},E)\phi(\vec{r},E)dE$ and so on …

Scalar

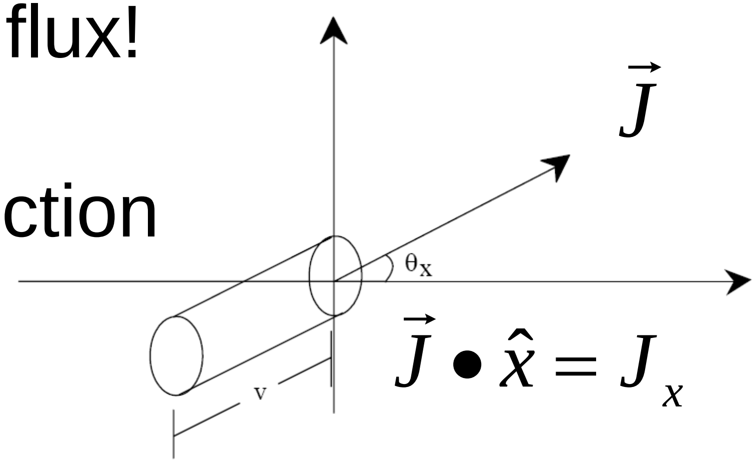
- Redefine $dI(\vec{r},\vec{\omega}) = n(\vec{r},\vec{\omega})v d\Omega$ as $d\vec{I}(\vec{r},\vec{\omega}) = n(\vec{r},\vec{\omega})\vec{v} d\Omega$

One group!

$$\phi(\vec{r}) = \int_{4\pi} v n(\vec{r},\vec{\omega}) d\Omega \quad \Rightarrow \quad \vec{J} = \int_{4\pi} \vec{v} n(\vec{r},\vec{\omega}) d\Omega$$

Neutron current density

- From larger flux to smaller flux!
- Neutrons are not pushed!
- More scattering in one direction than in the other.



$$\vec{J} \bullet \hat{x} = J_x$$

Equation of Continuity

Net flow of neutrons per second per unit area normal to the x direction:

$$\vec{J} \bullet \hat{x} = J_x = \int_{4\pi} n(\vec{r},\vec{\omega}) v \cos\theta_x d\Omega$$

In general: $\vec{J} \bullet \hat{n} = J_n$

Equation of Continuity

$$\frac{\partial}{\partial t}\int_{\forall} n(\vec{r},t) d\forall = \int_{\forall} S(\vec{r},t) d\forall - \int_{\forall} \Sigma_a(\vec{r})\phi(\vec{r},t) d\forall - \oint_A \vec{J}(\vec{r},t) \bullet \hat{n} dA$$

$\pm$

- Rate of change in number of neutrons
- Production rate
- Absorption rate
- "Leakage in/out" rate

Volume

Source distribution function

Surface area bounding $\forall$

Normal to A *(outwards)*

Equation of Continuity

Using Gauss' Divergence Theorem

$$\oint_S \vec{B} \bullet d\vec{A} = \int_V \vec{\nabla} \bullet \vec{B} d^3 r$$

Recall:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_A \vec{J}(\vec{r},t) \bullet \hat{n} dA = \int_\forall \vec{\nabla} \bullet \vec{J}(\vec{r},t) d\forall$$

$$\frac{\partial}{\partial t} \int_\forall n(\vec{r},t) d\forall = \int_\forall S(\vec{r},t) d\forall - \int_\forall \Sigma_a(\vec{r}) \phi(\vec{r},t) d\forall - \oint_A \vec{J}(\vec{r},t) \bullet \hat{n} dA$$

Both flux and current!!
Convert current to flux?

One group!

$$\frac{1}{v} \frac{\partial}{\partial t} \phi(\vec{r},t) = S(\vec{r},t) - \Sigma_a(\vec{r}) \phi(\vec{r},t) - \vec{\nabla} \bullet \vec{J}(\vec{r},t)$$

Fundamental equation in Reactor Theory

Equation of Continuity

Equation of Continuity

Steady state

$$S(\vec{r}) - \Sigma_a(\vec{r})\phi(\vec{r}) - \vec{\nabla} \bullet \vec{J}(\vec{r}) = 0$$

Non-spacial dependence

$$\frac{\partial}{\partial t} n(t) = S(t) - \Sigma_a \phi(t)$$

Delayed sources? Will do it later.