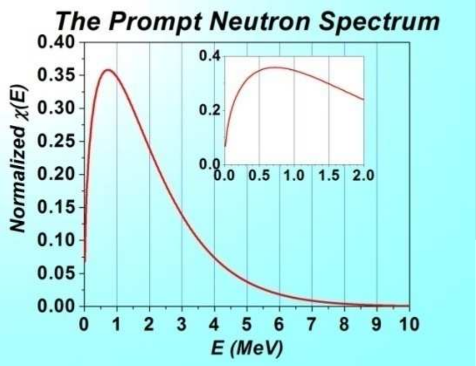
Controlled Fission

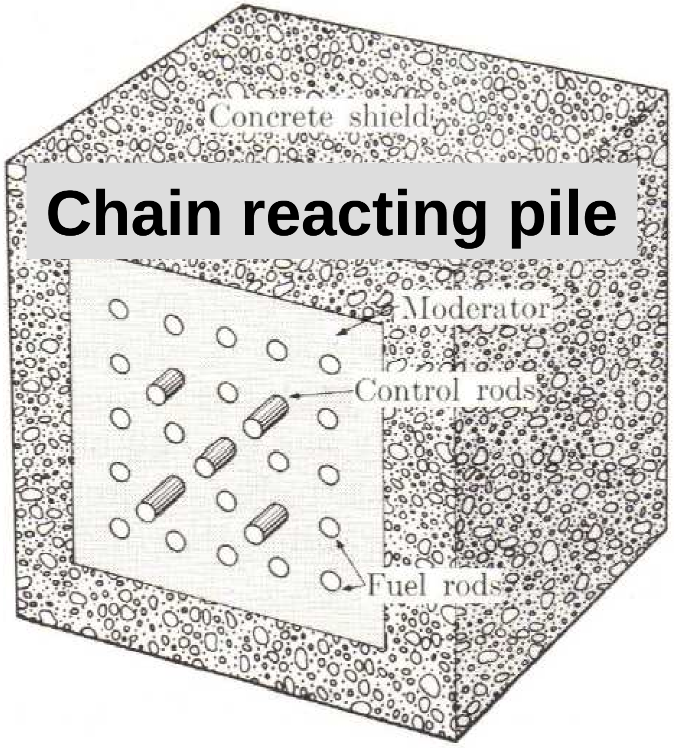


- $^{235}U + n \rightarrow X + Y + \overset{\nu}{(\sim 2.4)}n$ ← Fast second generation neutrons
- Moderation of second generation neutrons ▶ Chain reaction.
- Water, D_2O or graphite moderator.
- Ratio of number of "neutrons" (fissions) in one generation to the preceding $\equiv k_\infty$ (neutron reproduction or multiplication factor).

Infinite medium (ignoring leakage at the surface).

- $k \geq 1$ ▶ Chain reaction.
- $k < 1$ ▶ subcritical.
- $k = 1$ ▶ critical system.
- $k > 1$ ▶ supercritical.

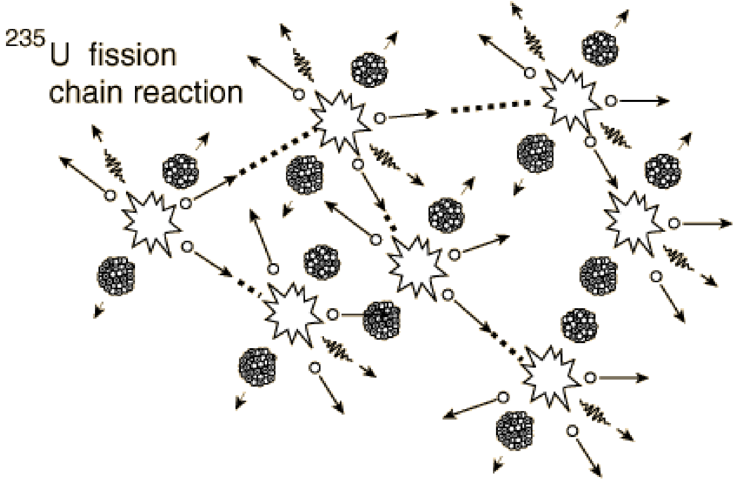
For steady release of energy (steady-state operation) we need $k = 1$.

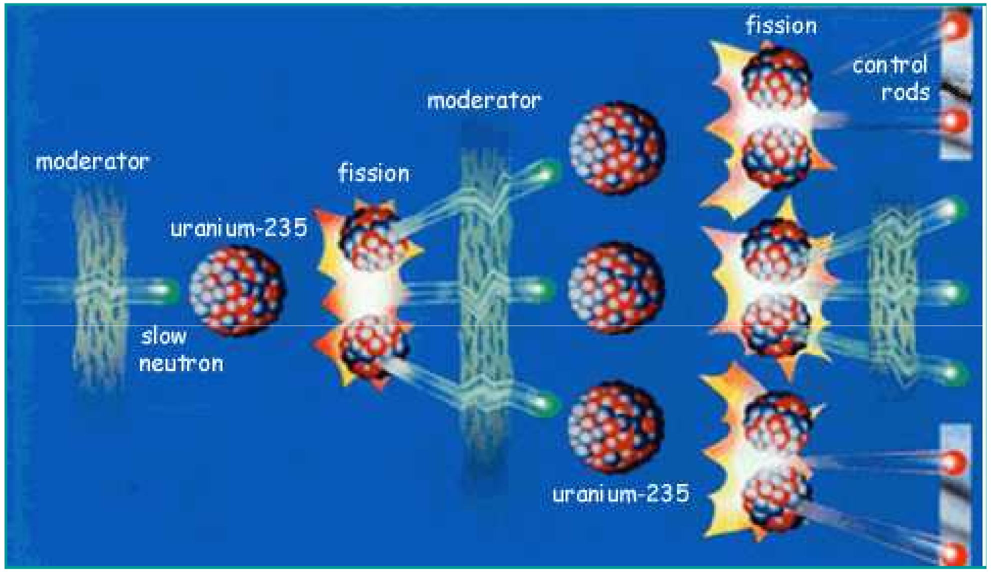

Chain reacting pile

Controlled Fission

- **Average number of all neutrons released per fission → ν (for thermal neutrons, 0.0253 eV).**
- **^{233}U : 2.492**
- **^{235}U : 2.418**
- **^{239}Pu : 2.871**
- **^{241}Pu : 2.927**

Fissile





Not all ν neutrons will subsequently cause fission…!

- **Reactor is critical ($k = 1$): rate of neutrons produced by fission = rate of neutrons *absorbed* + leaked.**

Size and composition of the reactor.

Controlled Fission

^{235}U *thermal cross sections*

$\sigma_{\text{fission}} \approx 584$ b.

$\sigma_{\text{scattering}} \approx 9$ b.

$\sigma_{\text{radiative capture}} \approx 97$ b.

Check numbers!

Probability for a thermal neutron to cause fission on ^{235}U is

$$\approx \frac{\sigma_f}{\sigma_f + \sigma_\gamma} = \frac{1}{1+\alpha}$$

Not all ν neutrons will subsequently cause fission…!

If each fission produces an average of ν neutrons, then the mean number of **fast** fission neutrons produced per thermal neutron = η

$$\boxed{\eta = \nu \frac{\sigma_f}{\sigma_a} = \nu \frac{\sigma_f}{\sigma_f + \sigma_\gamma} = \frac{\nu}{1+\alpha}}$$

$$\eta < \nu$$

Controlled Fission

- ***Assume natural uranium:***

99.2745% ^{238}U, 0.7200% ^{235}U.

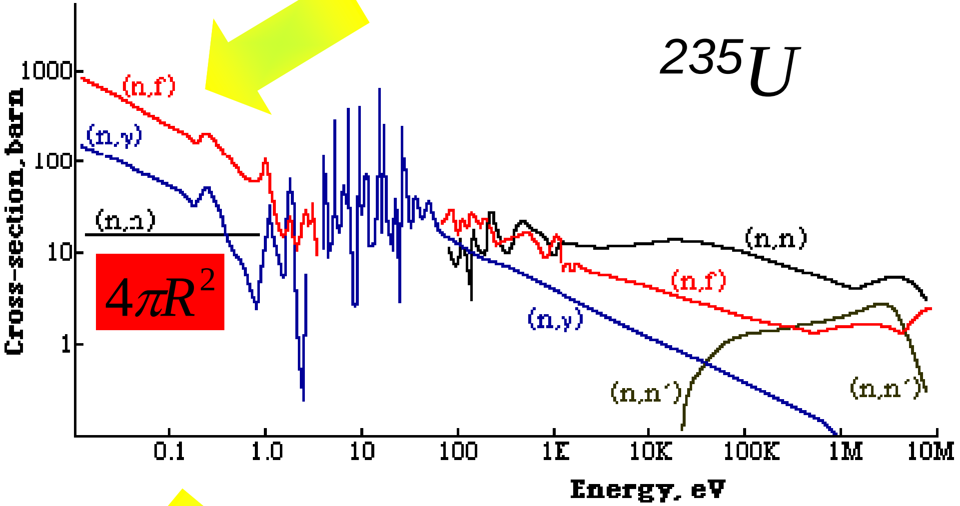
Thermal $\sigma_f = 0$ b   Why?   584 b

Thermal $\sigma_\gamma = 2.75$ b   97 b

$$\Sigma = \Sigma_x + \Sigma_y = N_x\sigma_x + N_y\sigma_y$$

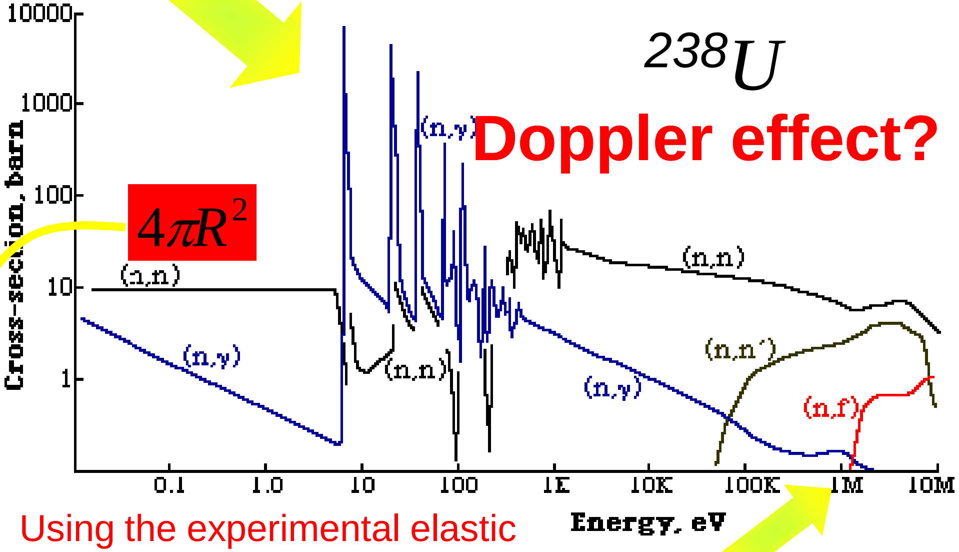
$$= (\gamma_x\sigma_x + \gamma_y\sigma_y)N$$

- Σ_f / N = (0.992745)(0) + (0.0072)(584) = 4.20 b.
- Σ_γ / N = (0.992745)(2.75) + (0.0072)(97) = 3.43 b.



^{235}U

$4\pi R^2$



^{238}U

Doppler effect?

$4\pi R^2$

Using the experimental elastic scattering data the radius of the nucleus can be estimated.

Controlled Fission

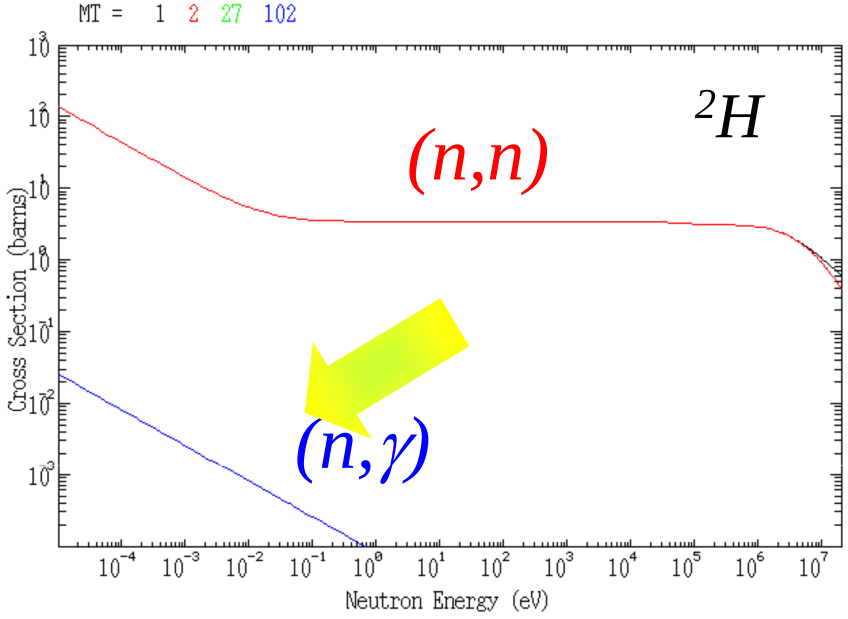
- Probability for a thermal neutron to cause fission in **natural uranium**

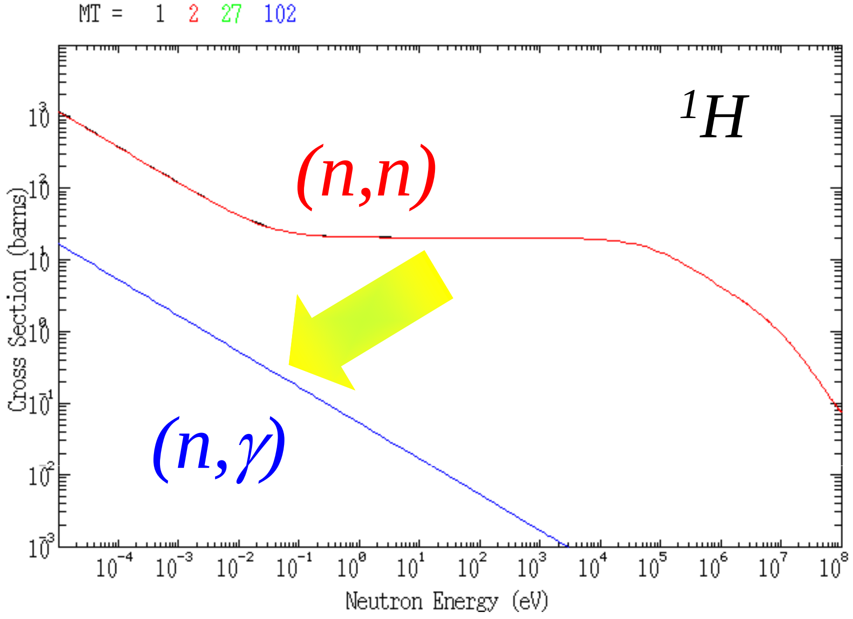
$$= \frac{4.20}{4.20 + 3.43} = 0.55$$

Compare to pure ^{235}U *and to 3% enriched fuel.*

- If each fission produces an average of ν = 2.4 neutrons, then the mean number of fast fission neutrons produced per thermal neutron = η = 2.4 x 0.55 $\approx$ 1.3
- This is close to 1. If neutrons are still to be lost, there is a danger of losing criticality. (Heavy water?).
- For **enriched uranium** (^{235}U = 3%) η = ????? (> 1.3). (Light water?).
- In this case η is further from 1 and allowing for more neutrons to be lost while maintaining criticality.

Moderation (to compare x-section)





Timeout..!

- Resonances?
- ^{3}H production.

Controlled Fission

HW 11

- Verify

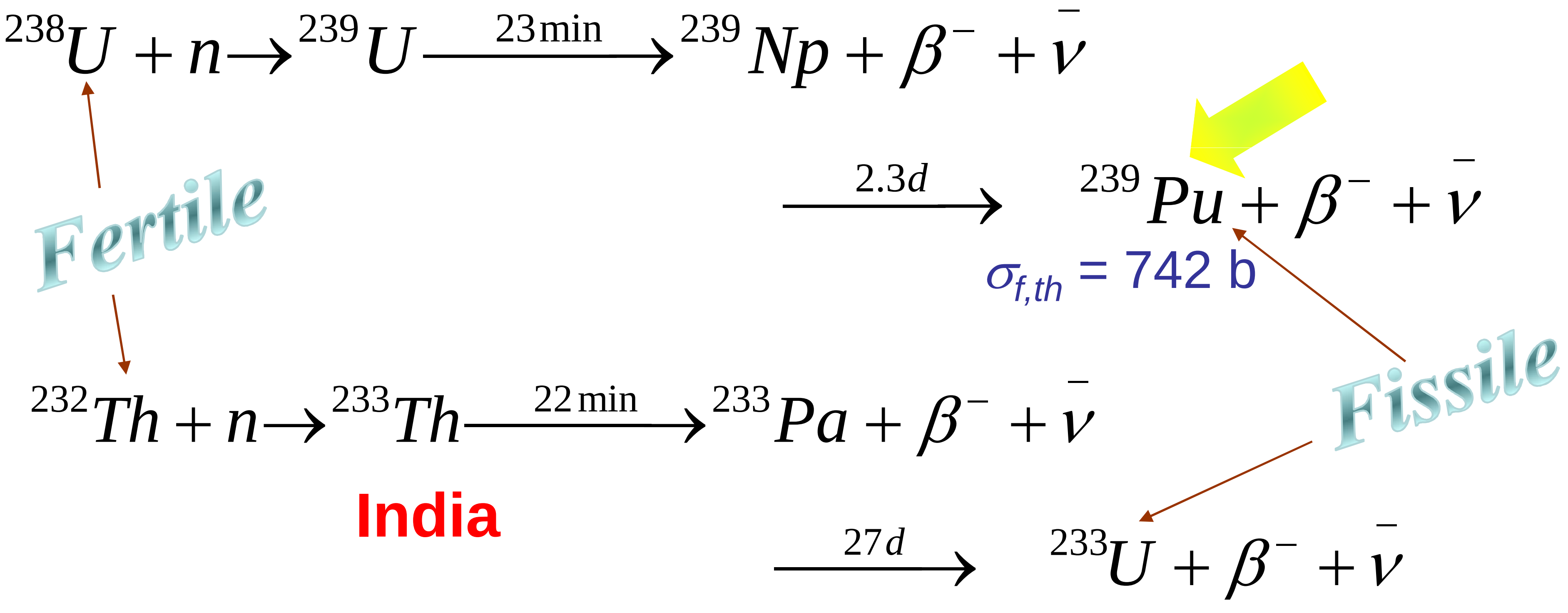
$$\eta = \frac{1}{\Sigma_a} \sum_i \nu(i) \Sigma_f(i)$$

- Comment on the calculation for thermal neutrons and a mixture of fissile and non-fissile materials, giving an example.
- Comment for fast neutrons and a mixture of fissionable materials, giving an example.

Conversion and Breeding

Timeout..!

Converters: Convert non-thermally-fissionable material to a thermally-fissionable material.



$$^{238}U + n \rightarrow {}^{239}U \xrightarrow{23\,\mathrm{min}} {}^{239}Np + \beta^- + \bar{\nu}$$

$$\xrightarrow{2.3d} {}^{239}Pu + \beta^- + \bar{\nu}$$

$\sigma_{f,th}$ = 742 b

Fertile

Fissile

$$^{232}Th + n \rightarrow {}^{233}Th \xrightarrow{22\,\mathrm{min}} {}^{233}Pa + \beta^- + \bar{\nu}$$

India

$$\xrightarrow{27d} {}^{233}U + \beta^- + \bar{\nu}$$

$\sigma_{f,th}$ = 530 b

Conversion and Breeding

Timeout..!

Delicate neutron economy…!

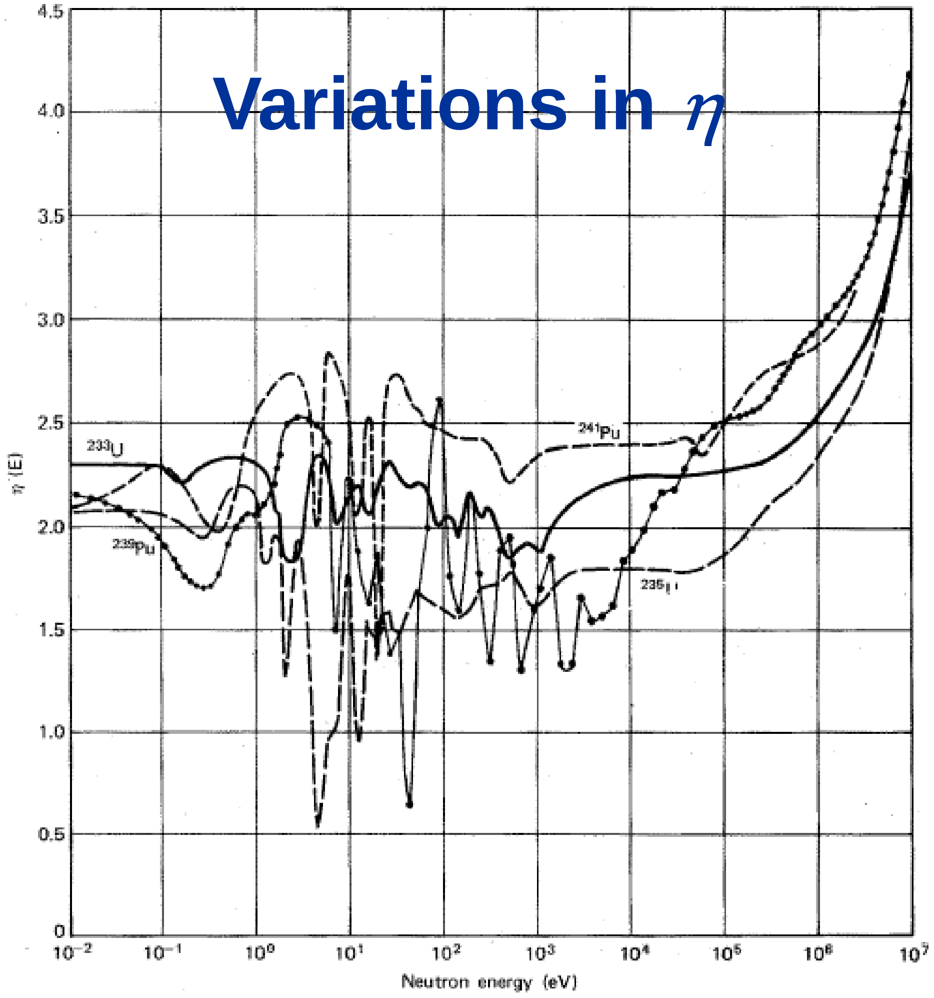
- If $\eta = 2$ ▶ Conversion and fission.
- If $\eta > 2$ ▶ Breeder reactor.
- ^{239}Pu: Thermal neutrons ($\eta \sim 2.1$) ▶ hard for breeding.
  Fast neutrons ($\eta \sim 3$) ▶ possible breeding ▶ fast breeder reactors.

- After sufficient time of breeding, fissile material can be easily (chemically) separated from fertile material.
  Compare to separating ^{235}U from ^{238}U.
- Reprocessing.

Controlled Fission

Timeout..!

- Note that η is greater than 2 at thermal energies and almost 3 at high energies.
- These “extra” neutrons are Used to convert fertile into fissile fuel.
- Plutonium economy.
- India and thorium.
- Efficiency of this process is determined by neutron energy spectrum.



Variations in η

Controlled Fission

Timeout..!

- **Conversion ratio** CR is defined as the average rate of fissile atom production to the average rate of fissile atom consumption.
- For LWR's CR $\cong$ 0.6.
- CR is called BR for values > 1.
- Fast breeder reactors have BR > 1.
- They are called “fast” because primary fissions inducing neutrons are fast not thermal, thus $\eta > 2.5$ but σ_f is only a few barns.
- Moderator??

Controlled Fission

- N **thermal** neutrons in one generation **have produced so far ηN fast neutrons.**
- Some of these **fast** neutrons can cause ^{238}U fission ▶ more fast neutrons ▶ **fast fission factor** = ε (= 1.03 for natural uranium).
- **Now we have $\varepsilon\eta N$ fast neutrons.**
- We need to moderate these fast neutrons ▶ use graphite as an example ▶ for 2 MeV neutrons we need ??? collisions. How many for 1 MeV neutrons?
- The neutron will pass through the 10 - 100 eV region during the moderation process. This energy region has many **strong** ^{238}U capture resonances (up to ????? b) ▶ Can not mix uranium and moderator.
- In graphite, an average distance of 19 cm is needed for thermalization ▶ the **resonance escape probability** p ($\approx$ 0.9).

Controlled Fission

- **Now we have $p\varepsilon\eta N$ thermal neutrons.**
- Moderator must not be too large to capture thermal neutrons; **when thermalized, neutrons should have reached the fuel.**
- Graphite thermal cross section = 0.0034 b, but there is a lot of it present.
- Capture can also occur in the material encapsulating the fuel elements (clad).
- The **thermal utilization factor** f ($\approx$ 0.9) gives the fraction of thermal neutrons that are actually available for the fuel.
- **Now we have $fp\varepsilon\eta N$ thermal neutrons**, could be > or < N thus determining the criticality of the reactor.

$$k_\infty = fp\varepsilon\eta$$

The four-factor formula.

$$k = k_{eff} = fp\varepsilon\eta(1-l_{fast})(1-l_{thermal})$$

Fractions lost at surface

Controlled Fission

$$k_\infty = fp\varepsilon\eta, \qquad k_{eff} = f\rho\varepsilon\eta P_{non-leak}$$

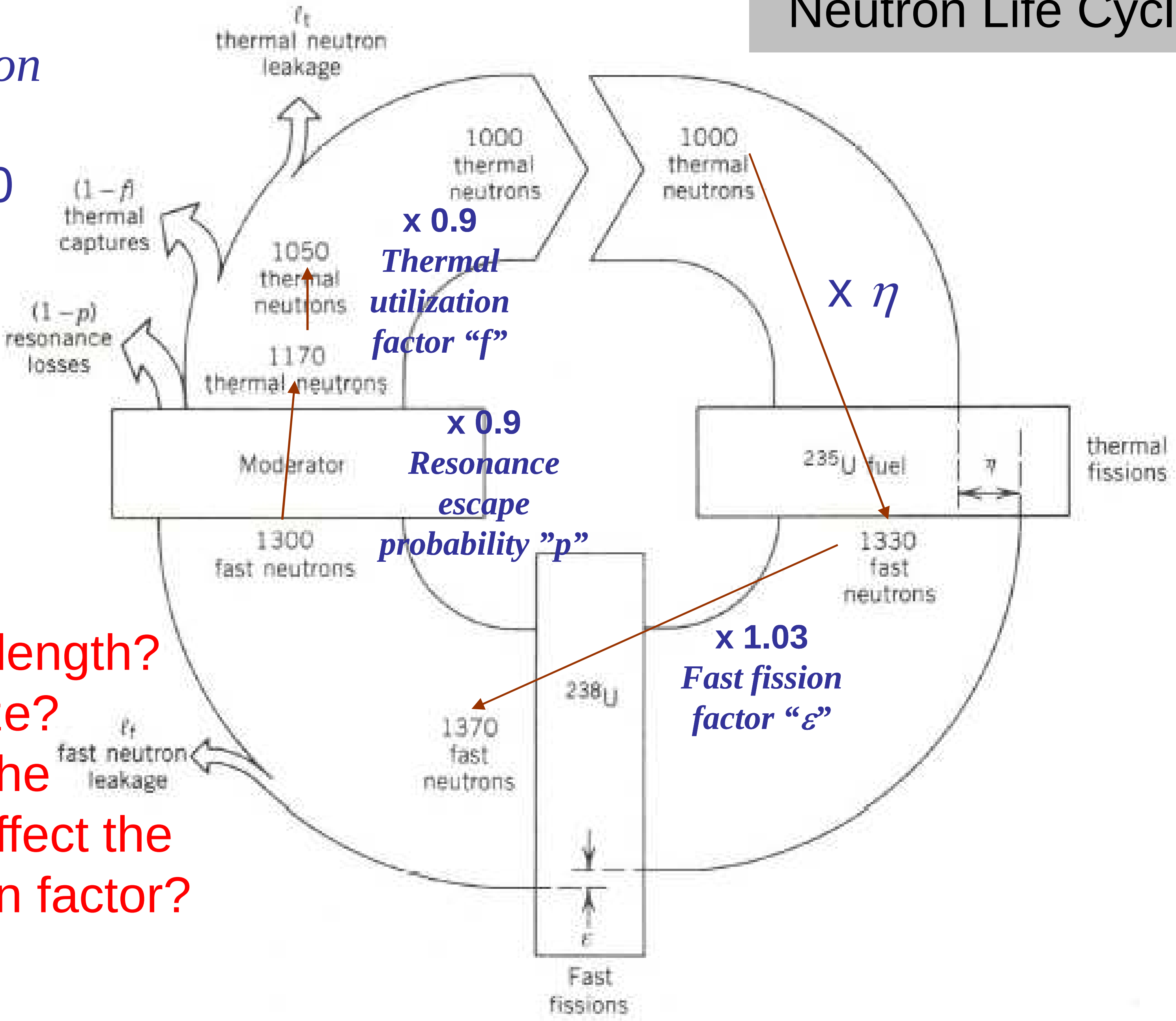
- Fast from thermal, $\eta = \frac{1}{\Sigma_a}\sum_i \nu(i)\Sigma_f(i)$ as defined in HW 11.
- Fast from fast, ε.
- Thermal from fast, p.
- Thermal available for fuel $f = \frac{\sum_a^{fuel}}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{\mathrm{mod}erator} + \sum_a^{rods} + \sum_a^{poison} + ..}$

Thinking QUIZ

- For each thermal neutron absorbed, how many fast neutrons are produced? Will need this when discuss two-group diffusion.

Neutron Life Cycle

Neutron reproduction factor
$k = 1.000$



x 0.9
Thermal utilization factor "f"

x 0.9
Resonance escape probability "p"

x 1.03
Fast fission factor "ε"

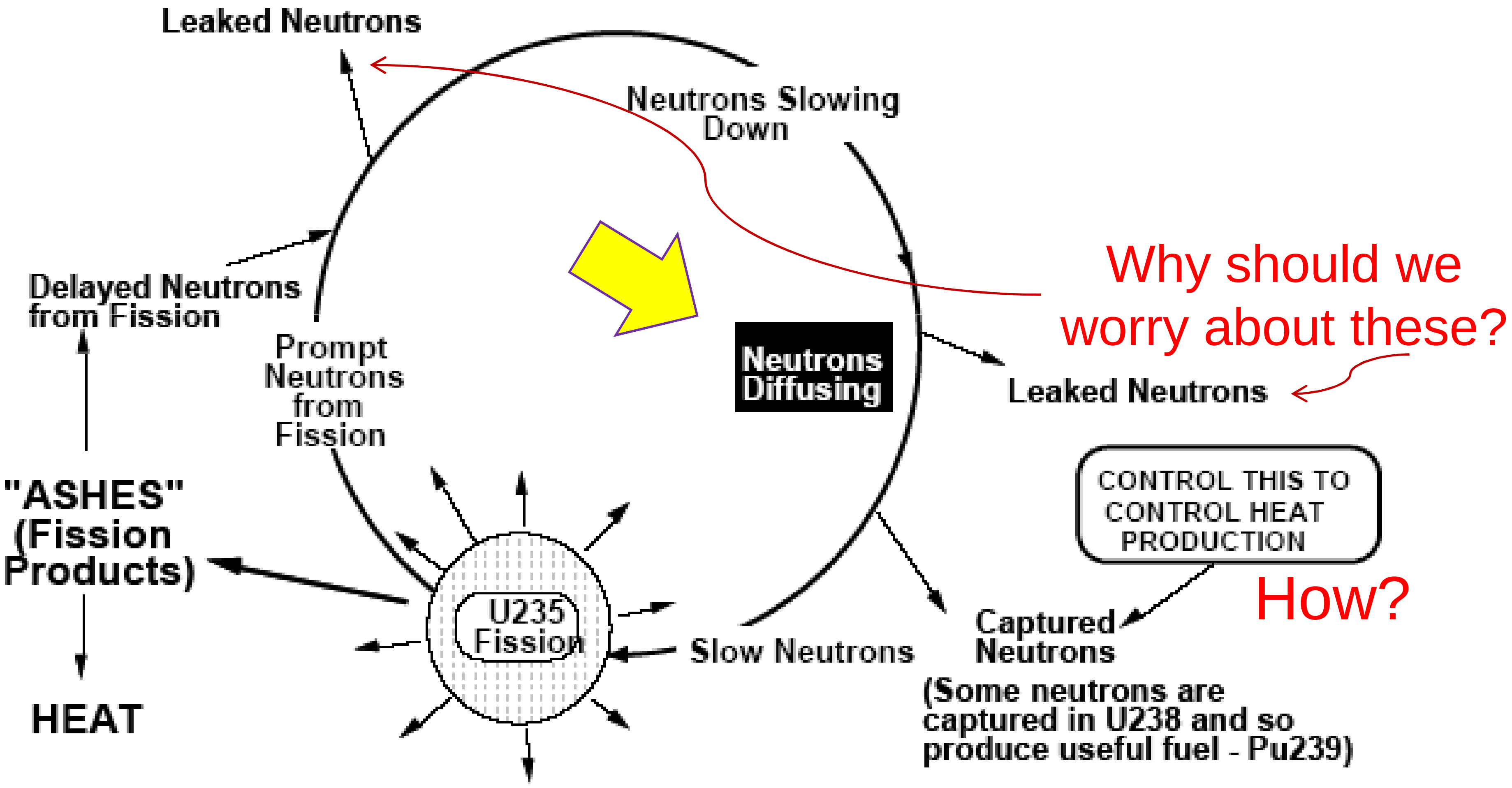
x η

What is:

- Migration length?
- Critical size?

How does the geometry affect the reproduction factor?

Neutron Life Cycle



Why should we worry about these?

How?

Controlled Fission

$k = fp\varepsilon\eta(1\text{-}l_{fast})(1\text{-}l_{thermal})$ Not fixed…!

- Thermal utilization factor f can be changed, as an example, by adding absorber to coolant (PWR) (chemical shim, boric acid), or
by inserting movable control rods in & out.
- Reactors can also be controlled by altering neutron leakages using movable neutron reflectors.
- f *and* p factors change as fuel is burned.
- f, p, η change as fertile material is converted to fissile material.

Controlled Fission

- Attention should be paid also to the fact that reactor power changes occur due to changes in resonance escape probability p. If Fuel T↑, p↓ *due to* Doppler broadening of resonance peaks.

Under-moderation
and
over-moderation.