Fuel Depletion

$$N \sim 10^{22}\,cm^{-3}, \phi \sim 10^{14}\,cm^{-2}s^{-1}$$

Time scale:
Days and months.

- More depletion ▶ increase steady state flux by means of reducing absorbers.
- For a given fuel isotope $\dfrac{\partial N_f(\vec{r},t)}{\partial t} = -N_f(\vec{r},t)\sigma_a^f \phi(\vec{r},t)$
- For **constant flux** ϕ_0 the solution is

Exponential burnup

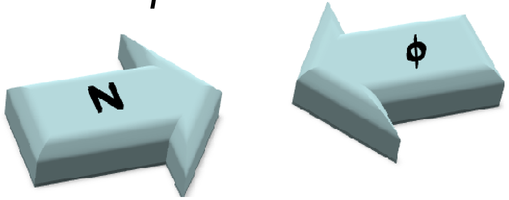
$$N_f(\vec{r},t) = N_f(\vec{r},0)e^{-\sigma_a^f \underbrace{\phi_0(\vec{r})t}} = N_f(\vec{r},0)e^{-\sigma_a^f \Phi(\vec{r},t)}$$

- For **time varying flux**

Neutron fluence

$$N_f(\vec{r},t) = N_f(\vec{r},0)e^{-\sigma_a^f \overbrace{\int_0^t \phi(\vec{r},t^\backslash)dt^\backslash}} = N_f(\vec{r},0)e^{-\sigma_a^f \Phi(\vec{r},t)}$$

Solve numerically.



Fuel Depletion

- **Constant power.**

$$P(\vec{r},t) = \underbrace{w}_{\text{Energy released per fission}}\underbrace{\overset{\downarrow}{N_f}(\vec{r},t)\sigma_f^f\overset{\uparrow}{\phi}(\vec{r},t)}_{\text{Fission rate}} = P(\vec{r},0) = P_0(\vec{r})$$

$$N_f(\vec{r},t)\phi(\vec{r},t) = N_f(\vec{r},0)\phi(\vec{r},0)$$

$$\Sigma_f(\vec{r},t)\phi(\vec{r},t) = \Sigma_f(\vec{r},0)\phi(\vec{r},0)$$

- Power ~ flux only over short time periods during which N_f is constant.

$$\frac{\partial N_f(\vec{r},t)}{\partial t} = -\underset{\downarrow}{N_f}(\vec{r},t)\sigma_a^f\underset{\uparrow}{\phi}(\vec{r},t) \approx -\frac{P_0(\vec{r})}{w}$$

Linear depletion!

$$\sigma_f^f \approx \sigma_a^f$$

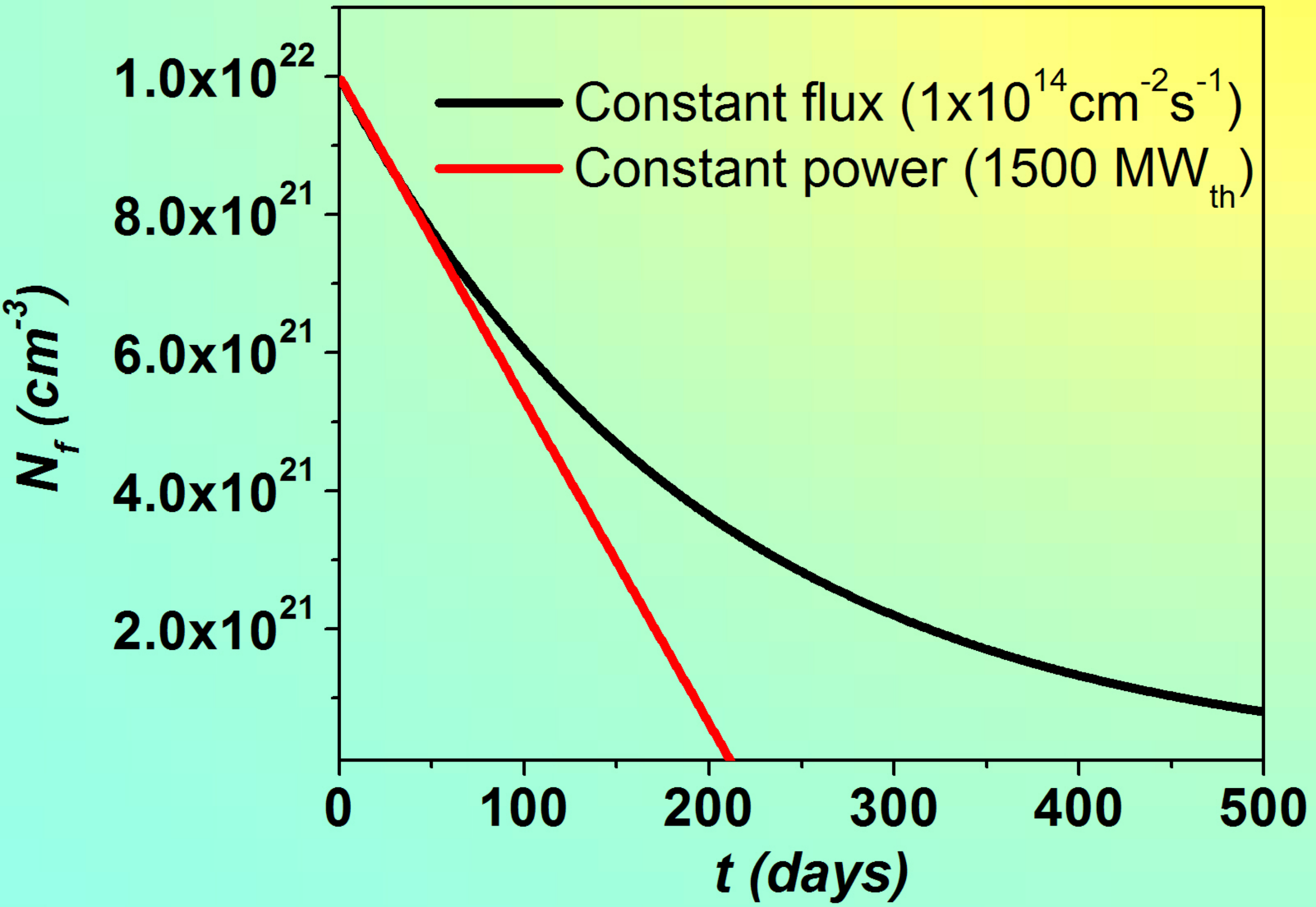
- The solution is obviously

$$N_f(\vec{r},t) \approx N_f(\vec{r},0) - \frac{P_0(\vec{r})}{w}t$$

Fuel Depletion

HW 31

Do the calculations for different flux and power levels.



Poisoning and Fuel Depletion

Infinite, critical homogeneous reactor.

$$k_\infty = \varepsilon\eta f\rho = \varepsilon\eta\rho\frac{\overset{\downarrow}{\Sigma_a^f(t)}}{\underset{\downarrow}{\Sigma_a^f(t)}+\Sigma_a^{clad}+\Sigma_a^{\text{moderator}}(t)+\underset{\uparrow}{\Sigma_a^{poison}(t)}+\underset{\text{thus}\downarrow}{\Sigma_a^{control}(t)}}$$

Constant power $N_f(\vec{r},t) \approx N_f(\vec{r},0) - \frac{P_0(\vec{r})}{w}t$

$$\phi(\vec{r},t) = \frac{N_f(\vec{r},0)}{N_f(\vec{r},t)}\phi(\vec{r},0) = \frac{\phi(\vec{r},0)}{1-\sigma_a^f\phi(\vec{r},0)t}$$

$$= N_f(\vec{r},0) - N_f(\vec{r},t)\sigma_a^f\phi(\vec{r},t)t$$

$$= N_f(\vec{r},0) - N_f(\vec{r},0)\sigma_a^f\phi(\vec{r},0)t$$

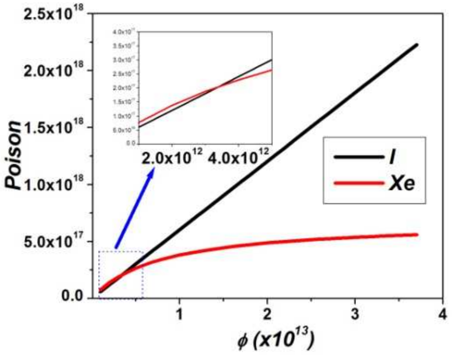
$$= N_f(\vec{r},0)\left[1-\sigma_a^f\phi(\vec{r},0)t\right]$$

$$\Sigma_a^f(\vec{r},t) = \Sigma_a^f(\vec{r},0)\left[1-\sigma_a^f\phi(\vec{r},0)t\right]$$

Poisoning and Fuel Depletion

$$Xe(t)=\overbrace{\frac{(\gamma_I+\gamma_{Xe})\overbrace{\Sigma_f\,\phi_0}^{\longrightarrow\ \text{Constant}}}{\lambda_{Xe}+\sigma_a^{Xe}\phi_0}}^{Xe_\infty}\left(1-e^{-(\lambda_{Xe}+\sigma_a^{Xe}\phi_0)t}\right)$$

$$+\frac{\gamma_I\,\Sigma_f\,\phi_0}{\lambda_{Xe}-\lambda_I+\sigma_a^{Xe}\phi_0}\left(e^{-(\lambda_{Xe}+\sigma_a^{Xe}\phi_0)t}-e^{-\lambda_I t}\right)$$



$$\Sigma_a^{Xe}(\vec{r},t)=\sigma_a^{Xe}Xe_\infty(\vec{r},t)=\frac{(\gamma_I+\gamma_{Xe})\overbrace{\Sigma_f(\vec{r},0)\phi(\vec{r},0)}^{\text{Constant}}}{\dfrac{\lambda_{Xe}}{\sigma_a^{Xe}}+\phi(\vec{r},t)}$$

$$\Sigma_a^{Sm}(\vec{r},t)\approx\sigma_a^{Sm}\gamma_{Sm}\,\Sigma_f(\vec{r},0)\phi(\vec{r},0)t$$

$$\phi(\vec{r},t)=\frac{\phi(\vec{r},0)}{1-\sigma_a^f\phi(\vec{r},0)t}$$

- Other fission products (poisons) with less capture cross sections.

Poisoning and Fuel Depletion

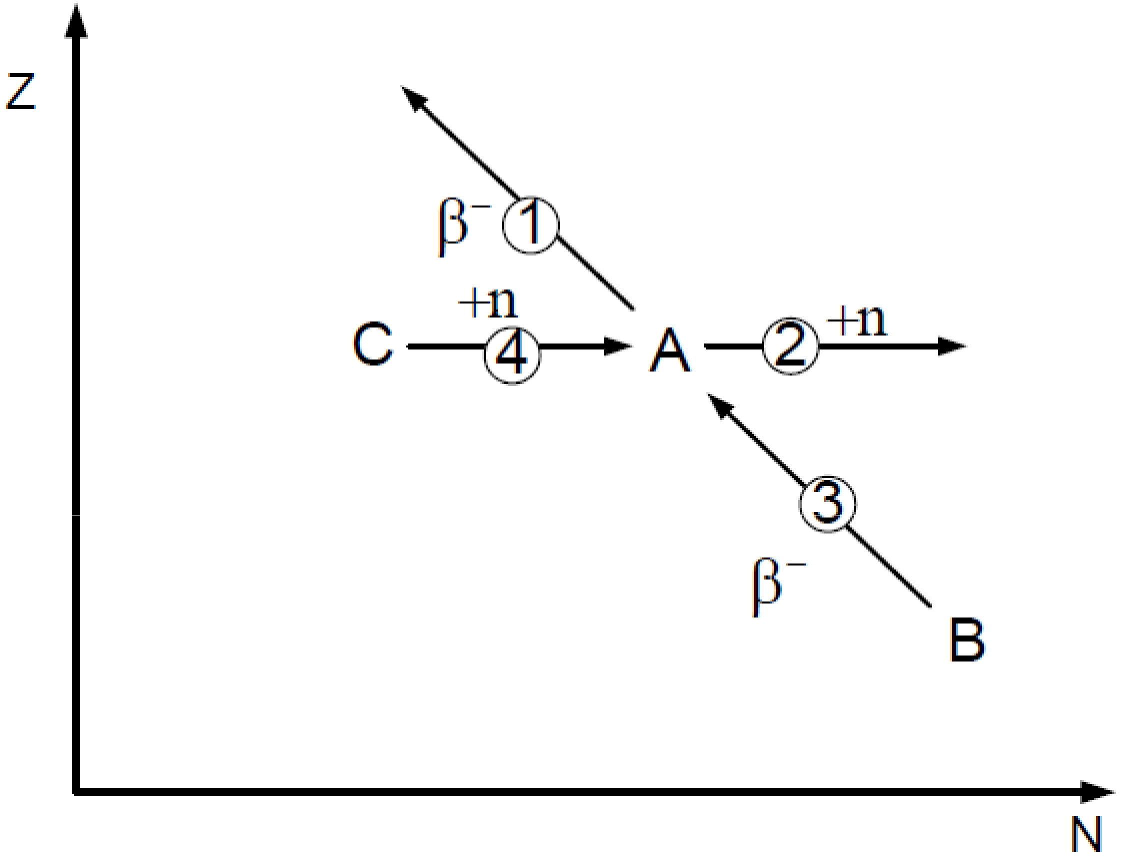
- Now we know all macroscopic cross sections.

$$k_\infty = \varepsilon\eta f\rho = \varepsilon\eta\rho \frac{\overset{\downarrow}{\sum_a^f (t)}}{\underset{\downarrow}{\sum_a^f (t)} + \sum_a^{clad} + \sum_a^{\text{moderator}} (t) + \underset{\uparrow}{\sum_a^{poison}} (t) + \sum_a^{control} (t)}$$

- When there are no absorbers left to remove, we need to refuel.
- Absorbers are not only control rods.
- All fuel nuclei should be considered.
- For each species, all sources and sinks should be taken into account.
- Online loading ▶ environmental.
- ^{3}H.

Until = 0.
Solve for t to get upper limit for “core loading lifetime”.
Damaged fuel…!

Poisoning and Fuel Depletion



$$\frac{dN_A}{dt} = -\lambda_A N_A - \phi\sigma_A N_A + \lambda_B N_B + \phi\sigma_\gamma^C N_C + F(t)$$

Fuel loading

Poisoning and Fuel Depletion

- **Some poisons are intentionally introduced into the reactor.**
  - Fixed burnable poisons.

    B, Gd.

    More uniform distribution than rods, more intentionally localized than shim.
  - Soluble poisons (chemical shim) with caution.

    Boric acid (soluble boron, solbor) in coolant.

    Boration and dilution.

    Scram emergency shutdown (sodium polyborate or gadolinium nitrate).
  - Non-burnable poisons.

    Chain of absorbers or self shielding.

Flatter flux.

Power shaping.

Delayed Precursors

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g^{\backslash}=1}^{G} \nu_{g^{\backslash}} \Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) + \sum_{g^{\backslash}=1}^{G} \Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) + S_g^{ext}$$

$$-\Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

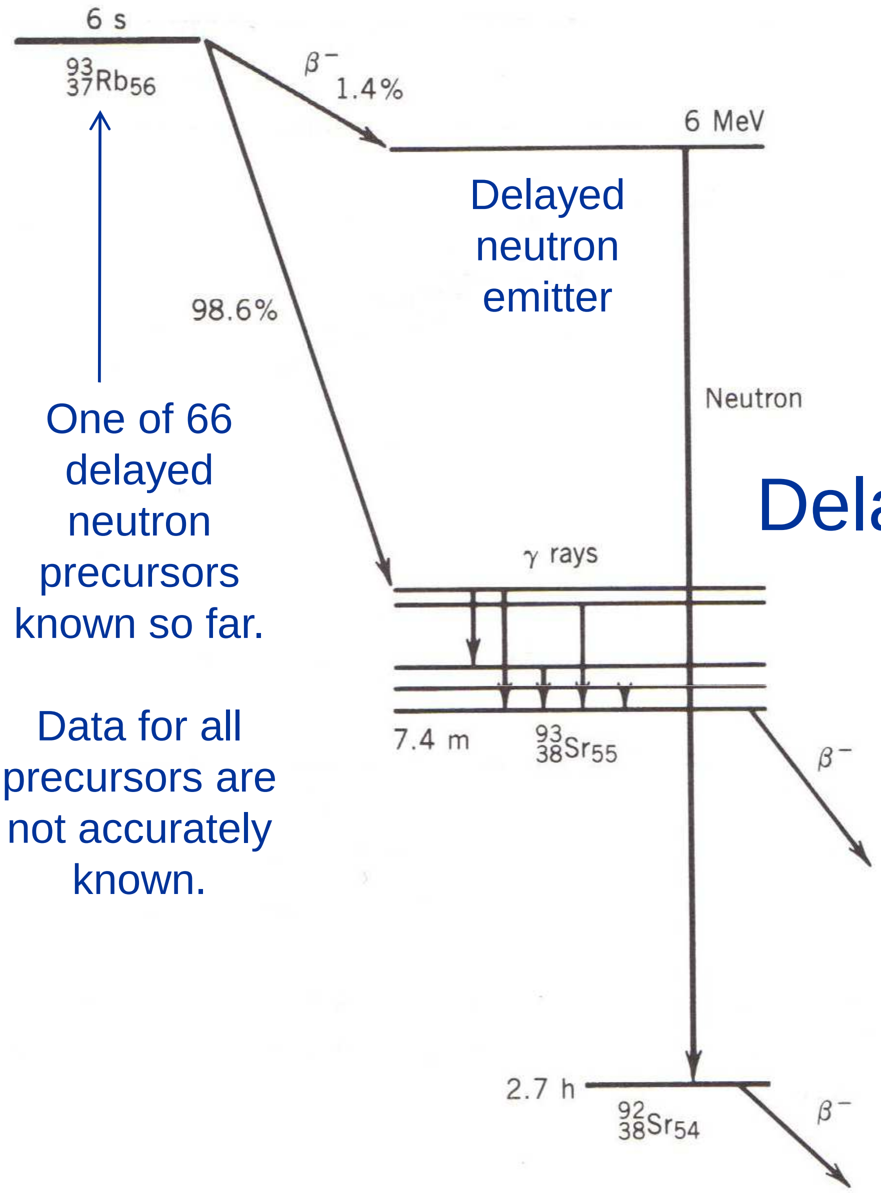
- For one-group

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = \nu\Sigma_f(\vec{r})\phi(\vec{r},t) + S^{ext}$$

$$-\Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla} \bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

- **What about delayed neutrons?**

Delayed Precursors



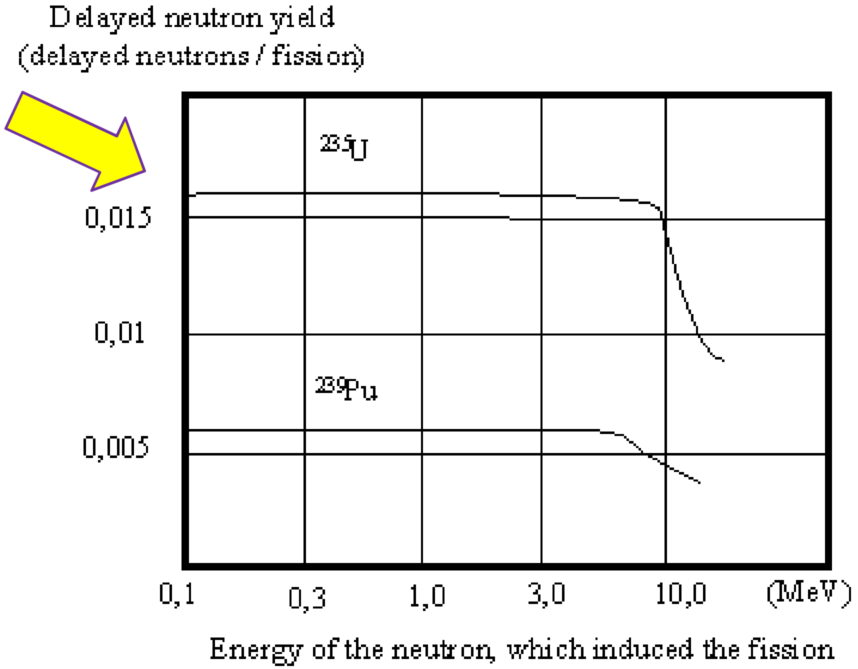
Delayed neutron emitter

One of 66 delayed neutron precursors known so far.

Data for all precursors are not accurately known.

$$\nu = \nu_p + \nu_d$$

Delayed neutron fraction $\beta = \dfrac{\nu_d}{\nu}$



Delayed Precursors

Fissile nucleus	Delayed neutron / 100 fissions
^{233}U	0.667
^{235}U	1.621
^{238}U*	4.39
^{239}Pu	0.628
^{240}Pu*	0.95
^{241}Pu	1.52
^{242}Pu*	2.21

Increases with *N*.

Data for thermal neutron induced fission, except for * fast neutron induced fission.

Delayed Precursors

Group #→	1	2	3	4	5	6
$T_{1/2}$ (s)	54.51	21.84	6.0	2.23	0.496	0.179
λ_I	0.0127	0.031	0.1155	0.310	1.397	3.871
β_i/β	.038	0.213	0.188	0.407	0.128	0.026
β_I	0.0002641	0.00148035	0.0013066	0.00282865	0.0008896	0.0001807

^{235}U

$\beta <$ 0.7% = 0.016 / ν

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = \overbrace{(1-\beta)\nu}^{\nu_p}\Sigma_f(\vec{r})\phi(\vec{r},t) + \sum_{i=1}^{6}\lambda_i C_i + S^{ext}$$

$$- \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla}\bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

$$\frac{\partial C_i(\vec{r},t)}{\partial t} = -\lambda_i C_i(\vec{r},t) + \beta_i \nu \Sigma_f(\vec{r})\phi(\vec{r},t)$$

Delayed Precursors

- The multi-group equation now becomes

Different energy spectra

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g^p(1-\beta)\sum_{g^{\backslash}=1}^{G}\nu_{g^{\backslash}}\Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) + \chi_g^C\sum_{i=1}^{6}\lambda_i C_i(\vec{r},t)$$

$$+\sum_{g^{\backslash}=1}^{G}\Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) + S_g^{ext}$$

$$-\Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

$$\frac{\partial C_i(\vec{r},t)}{\partial t} = -\lambda_i C_i(\vec{r},t) + \beta_i\sum_{g^{\backslash}=1}^{G}\nu_{g^{\backslash}}\Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t)$$

Full Blown Diffusion Equation

- In **steady state**

$$\lambda_i C_i(\vec{r},t) = \beta_i \sum_{g^{\backslash}=1}^{G} \nu_{g^{\backslash}} \Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t)$$

$$0 = \chi_g^{p}(1-\beta)\sum_{g^{\backslash}=1}^{G} \nu_{g^{\backslash}} \Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r}) + \chi_g^{C}\beta\sum_{g^{\backslash}=1}^{G} \nu_{g^{\backslash}} \Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r})$$

$$+\sum_{g^{\backslash}=1}^{G} \Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r}) + S_g^{ext} - \Sigma_{ag}(\vec{r})\phi_g(\vec{r}) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r}) + \vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r})$$

Significance of χ_g^{C} depends on whether we have fine or course energy groups.

$$0 = \left[\chi_g^{p} + (\chi_g^{C} - \chi_g^{p})\beta\right]\sum_{g^{\backslash}=1}^{G} \nu_{g^{\backslash}} \Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r})$$

$$+\sum_{g^{\backslash}=1}^{G} \Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r}) + S_g^{ext} - \Sigma_{ag}(\vec{r})\phi_g(\vec{r})$$

$$-\Sigma_{sg}(\vec{r})\phi_g(\vec{r}) + \vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r})$$