Multi-group Model

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$
$$- \Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

Calculate group-averaged:

$$\frac{1}{v_g}, \Sigma_{fg'}(\vec{r}), \Sigma_{sg'g}(\vec{r}), \Sigma_{ag}(\vec{r}), \Sigma_{sg}(\vec{r}), D_g(\vec{r})$$

Or for,

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$
$$- \Sigma_{rg}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sgg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

we need group-averaged $\Sigma_{rg}(\vec{r}), \Sigma_{sgg}(\vec{r})$

Multi-group Model

- *Group-averaged parameters?*
- *ENDF.*

$$\frac{1}{v(E)}\frac{\partial\phi(\vec{r},E,t)}{\partial t}=\chi(E)\int_0^\infty \nu(E')\Sigma_f(\vec{r},E')\phi(\vec{r},E',t)dE'$$

Units!

$$+\int_0^\infty \Sigma_s(\vec{r},E'\to E)\phi(\vec{r},E',t)dE' + S^{ext}$$

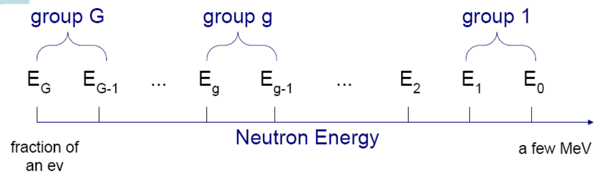
$$-\Sigma_a(\vec{r},E)\phi(\vec{r},E,t)-\Sigma_s(\vec{r},E)\phi(\vec{r},E,t)$$

$$+\vec{\nabla}\bullet D(\vec{r},E)\vec{\nabla}\phi(\vec{r},E,t)$$

- *Integrate term by term over groups and equate to equation of multi-group model.*

Multi-group Model

- Define group flux $\phi_g(\vec{r},t) \equiv \int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)dE$



$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \frac{\partial}{\partial t}\frac{\phi_g(\vec{r},t)}{v_g} = \frac{\partial}{\partial t}\int_{E_g}^{E_{g-1}} \frac{1}{v(E)}\phi(\vec{r},E,t)dE$$

$$\frac{1}{v_g} = \frac{\int_{E_g}^{E_{g-1}} \frac{1}{v(E)}\phi(\vec{r},E,t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)dE}$$

Multi-group Model

$$\vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t) = \vec{\nabla} \bullet \int_{E_g}^{E_{g-1}} D(\vec{r},E)\vec{\nabla}\phi(\vec{r},E,t)dE$$

▼

$$D_g(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} D(\vec{r},E)\vec{\nabla}\phi(\vec{r},E,t)dE}{\int_{E_g}^{E_{g-1}} \vec{\nabla}\phi(\vec{r},E,t)dE}$$

Multi-group Model

$$\Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) = \int_{E_g}^{E_{g-1}} \Sigma_a(\vec{r},E)\phi(\vec{r},E,t)dE$$

▼

$$\Sigma_{ag}(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} \Sigma_a(\vec{r},E)\phi(\vec{r},E,t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)dE}$$

Multi-group Model

$$\Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) = \int_{E_g}^{E_{g-1}} \Sigma_s(\vec{r},E)\phi(\vec{r},E,t)dE$$

▼

$$\Sigma_{sg}(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} \Sigma_s(\vec{r},E)\phi(\vec{r},E,t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)dE}$$

Multi-group Model

$$\sum_{g'=1}^{G}\Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t)=\int_{E_g}^{E_{g-1}}\int_{0}^{\infty}\Sigma_s(\vec{r},E'\rightarrow E)\phi(\vec{r},E',t)dE'dE$$

$$=\int_{E_g}^{E_{g-1}}\sum_{g'=1}^{G}\int_{E_{g'}}^{E_{g'-1}}\Sigma_s(\vec{r},E'\rightarrow E)\phi(\vec{r},E',t)dE'dE$$

$$=\sum_{g'=1}^{G}\int_{E_g}^{E_{g-1}}\int_{E_{g'}}^{E_{g'-1}}\Sigma_s(\vec{r},E'\rightarrow E)\phi(\vec{r},E',t)dE'dE$$

▼

$$\Sigma_{sg'g}(\vec{r})=\frac{1}{\phi_{g'}(\vec{r},t)}\int_{E_g}^{E_{g-1}}\int_{E_{g'}}^{E_{g'-1}}\Sigma_s(E'\rightarrow E)\phi(\vec{r},E',t)dE'dE$$

Multi-group Model

$$\chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) = \int_{E_g}^{E_{g-1}} \chi(E) \int_0^{\infty} \nu(E') \Sigma_f(E')\phi(\vec{r},E',t)dE' dE$$

$$= \int_{E_g}^{E_{g-1}} \chi(E)dE \int_0^{\infty} \nu(E') \Sigma_f(E')\phi(\vec{r},E',t)dE'$$

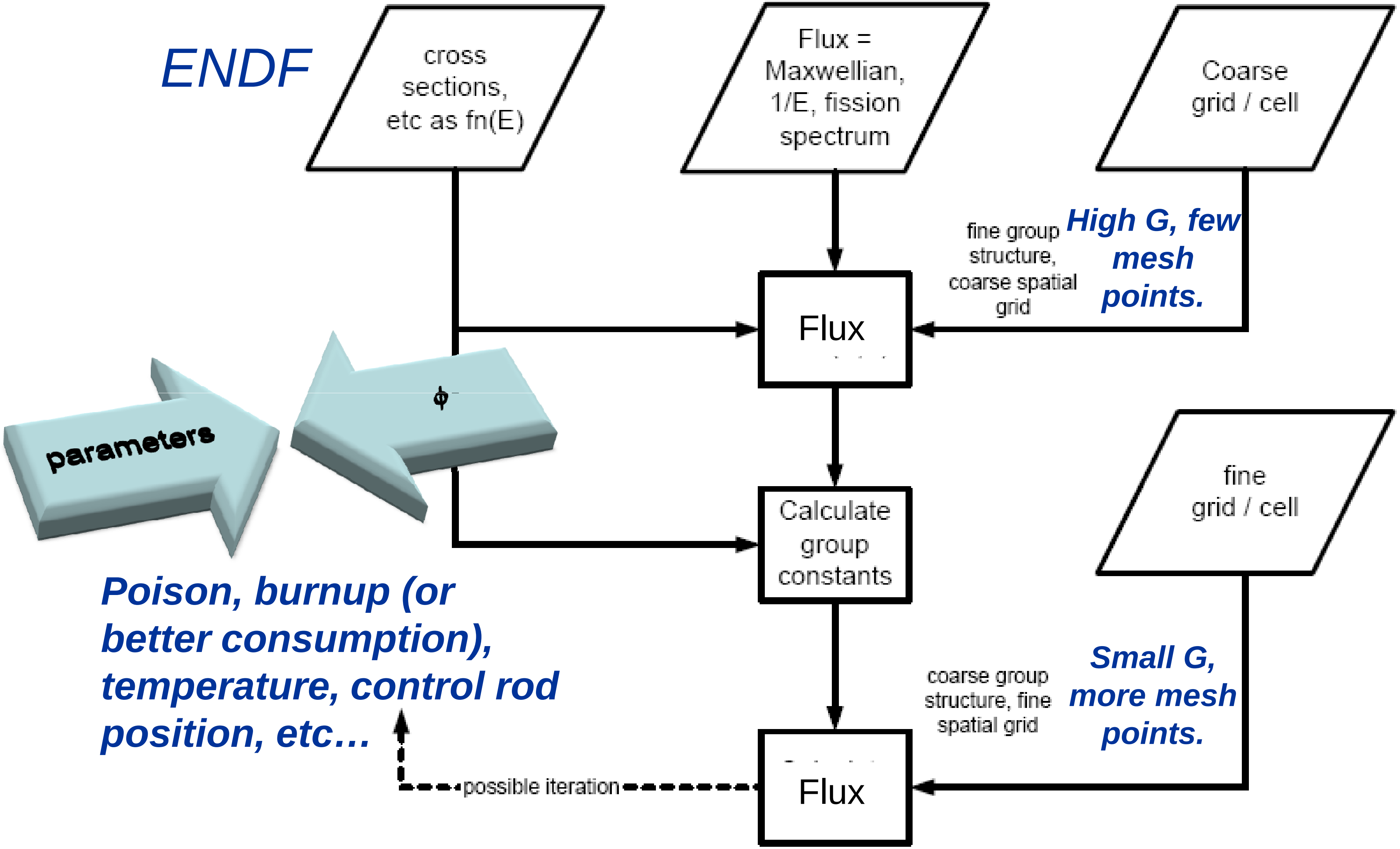
$$\chi_g = \int_{E_g}^{E_{g-1}} \chi(E)dE$$

$$= \chi_g \int_0^{\infty} \nu(E') \Sigma_f(E')\phi(\vec{r},E',t)dE'$$

$$= \chi_g \sum_{g'=1}^{G} \int_{E_{g'}}^{E_{g'=1}} \nu(E') \Sigma_f(E')\phi(\vec{r},E',t)dE'$$

$$\nu_{g'} \Sigma_{fg'}(\vec{r}) = \frac{1}{\phi_{g'}(\vec{r},t)} \int_{E_{g'}}^{E_{g'=1}} \nu(E') \Sigma_f(E')\phi(\vec{r},E',t)dE'$$

Multi-group Model



Multi-group Model

What could make life a little easier?!

- No upscattering $\Sigma_{sg^{\backslash}g}(\vec{r}) = 0 \text{ for } g^{\backslash} > g.$

  set group G to include neutrons up to ~1 eV.

$$\sum_{g^{\backslash}=1}^{G} \Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) \Rightarrow \sum_{g^{\backslash}=1}^{g-1} \Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) + \underbrace{\left(\Sigma_{sgg}(\vec{r})\phi_g(\vec{r},t)\right)}$$

Your choice of how to tackle in-scattering.

- No group skipping when scattering down (directly coupled).

$$\sum_{g^{\backslash}=1}^{G} \Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) \Rightarrow \Sigma_{s(g-1)g}(\vec{r})\phi_{g-1}(\vec{r},t) + \left(\Sigma_{sgg}(\vec{r})\phi_g(\vec{r},t)\right)$$

HW 27 How can we pledge this? What about H?

Multi-group Model

Criticality

Not all sources, only fission.

$$M\phi = \frac{1}{k} F\phi$$

Not only sinks

Iterations.

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{\substack{g'=1 \\ g' \neq g}}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$
$$- \Sigma_{rg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

No upscatter

$$-\vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t) + \Sigma_{rg}(\vec{r})\phi_g(\vec{r},t) - \sum_{\substack{g'=1 \\ g' \neq g}}^{g-1} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t)$$

Redundant when no upscatter.

$$= \frac{1}{K}\chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t)$$

Multi-group Model

No upscatter

$$M = \begin{bmatrix} -\vec{\nabla}\bullet D_1\vec{\nabla}+\Sigma_{r1} & 0 & 0 & \cdots \\ -\Sigma_{s12} & -\vec{\nabla}\bullet D_2\vec{\nabla}+\Sigma_{r2} & 0 & \cdots \\ -\Sigma_{s13} & -\Sigma_{s23} & -\vec{\nabla}\bullet D_3\vec{\nabla}+\Sigma_{r3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$F = \begin{bmatrix} \chi_1\nu_1\Sigma_{f1} & \chi_1\nu_2\Sigma_{f2} & \chi_1\nu_3\Sigma_{f3} & \cdots \\ \chi_2\nu_1\Sigma_{f1} & \chi_2\nu_2\Sigma_{f2} & \chi_2\nu_3\Sigma_{f3} & \cdots \\ \chi_3\nu_1\Sigma_{f1} & \chi_3\nu_2\Sigma_{f2} & \chi_3\nu_3\Sigma_{f3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \end{bmatrix}$$

Iterations.

Multi-group Model

No upscatter

$$M = \begin{bmatrix} -\vec{\nabla}\bullet D_1\vec{\nabla}+\Sigma_{r1} & 0 & 0 & \cdots \\ -\Sigma_{s12} & -\vec{\nabla}\bullet D_2\vec{\nabla}+\Sigma_{r2} & 0 & \cdots \\ 0 & -\Sigma_{s23} & -\vec{\nabla}\bullet D_3\vec{\nabla}+\Sigma_{r3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Directly coupled

$$F = \begin{bmatrix} \chi_1\nu_1\Sigma_{f1} & \chi_1\nu_2\Sigma_{f2} & \chi_1\nu_3\Sigma_{f3} & \cdots \\ \chi_2\nu_1\Sigma_{f1} & \chi_2\nu_2\Sigma_{f2} & \chi_2\nu_3\Sigma_{f3} & \cdots \\ \chi_3\nu_1\Sigma_{f1} & \chi_3\nu_2\Sigma_{f2} & \chi_3\nu_3\Sigma_{f3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \end{bmatrix}$$

Iterations.

Multi-group Model

Multi-group ⇒ one-group

$$\phi_g(\vec{r},t) \equiv \int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)dE \Rightarrow \phi(\vec{r},t) \equiv \int_0^{\infty} \phi(\vec{r},E,t)dE$$

$$\frac{1}{v_g} = \frac{\displaystyle\int_{E_g}^{E_{g-1}} \frac{1}{v(E)}\phi(\vec{r},E,t)dE}{\displaystyle\int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)dE} \Rightarrow \frac{1}{v} = \frac{\displaystyle\int_0^{\infty} \frac{1}{v(E)}\phi(\vec{r},E,t)dE}{\displaystyle\int_0^{\infty} \phi(\vec{r},E,t)dE}$$

Multi-group Model

$$D_g(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} D(\vec{r},E)\vec{\nabla}\phi(\vec{r},E,t)dE}{\int_{E_g}^{E_{g-1}} \vec{\nabla}\phi(\vec{r},E,t)dE} \Rightarrow D(\vec{r}) = \frac{\int_0^\infty D(\vec{r},E)\vec{\nabla}\phi(\vec{r},E,t)dE}{\int_0^\infty \vec{\nabla}\phi(\vec{r},E,t)dE}$$

$$\Sigma_{ag}(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} \Sigma_a(\vec{r},E)\phi(\vec{r},E,t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)dE} \Rightarrow \Sigma_a(\vec{r}) = \frac{\int_0^\infty \Sigma_a(\vec{r},E)\phi(\vec{r},E,t)dE}{\int_0^\infty \phi(\vec{r},E,t)dE}$$

Multi-group Model

$$\sum_{g^{\backslash}=1}^{G} \Sigma_{sg^{\backslash}g}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_{g}(\vec{r},t) \Rightarrow 0 \text{ when } G=1$$

$$\chi_g = \int_{E_g}^{E_{g-1}} \chi(E)dE \Rightarrow \chi = \int_{0}^{\infty} \chi(E)dE = 1$$

$$\sum_{g^{\backslash}=1}^{G} \nu_{g^{\backslash}} \Sigma_{fg^{\backslash}}(\vec{r})\phi_{g^{\backslash}}(\vec{r},t) \Rightarrow \nu \Sigma_f(\vec{r})\phi(\vec{r},t) \text{ when } G=1$$

Multi-group Model

Substituting all of the above into

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$

$$- \Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

yields

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = \nu \Sigma_f(\vec{r})\phi(\vec{r},t) + S^{ext}$$

$$- \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla} \bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

which is the one-group diffusion equation.

Multi-group Model

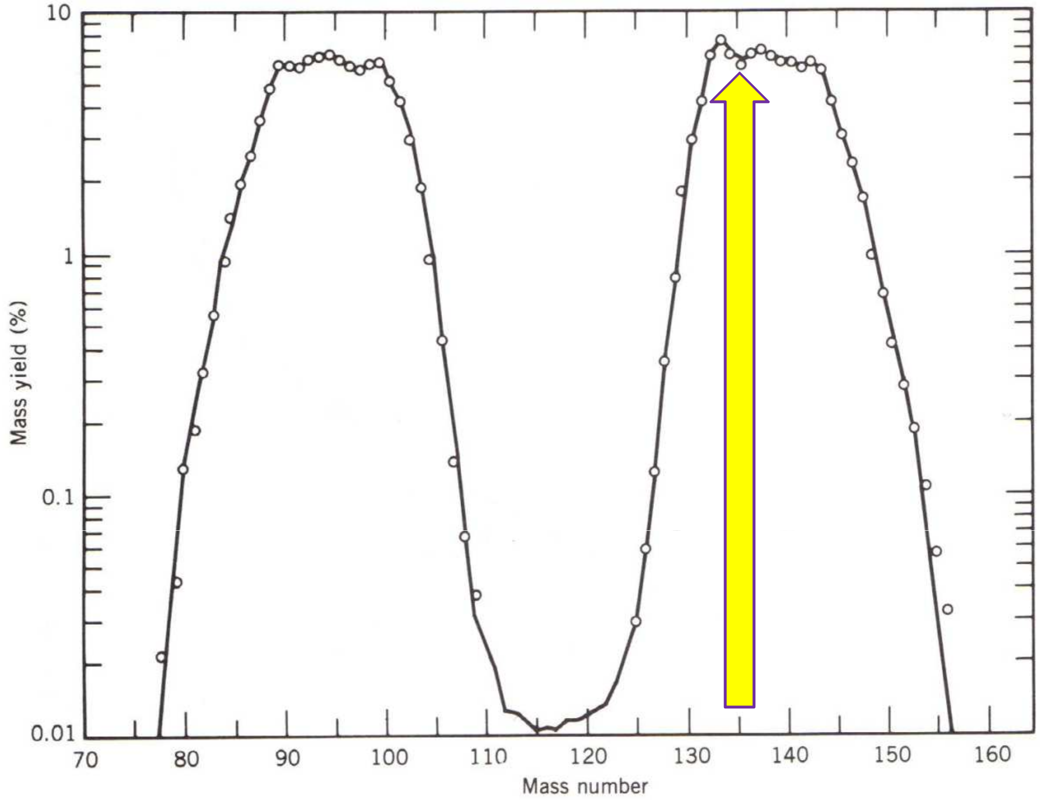
Project 3

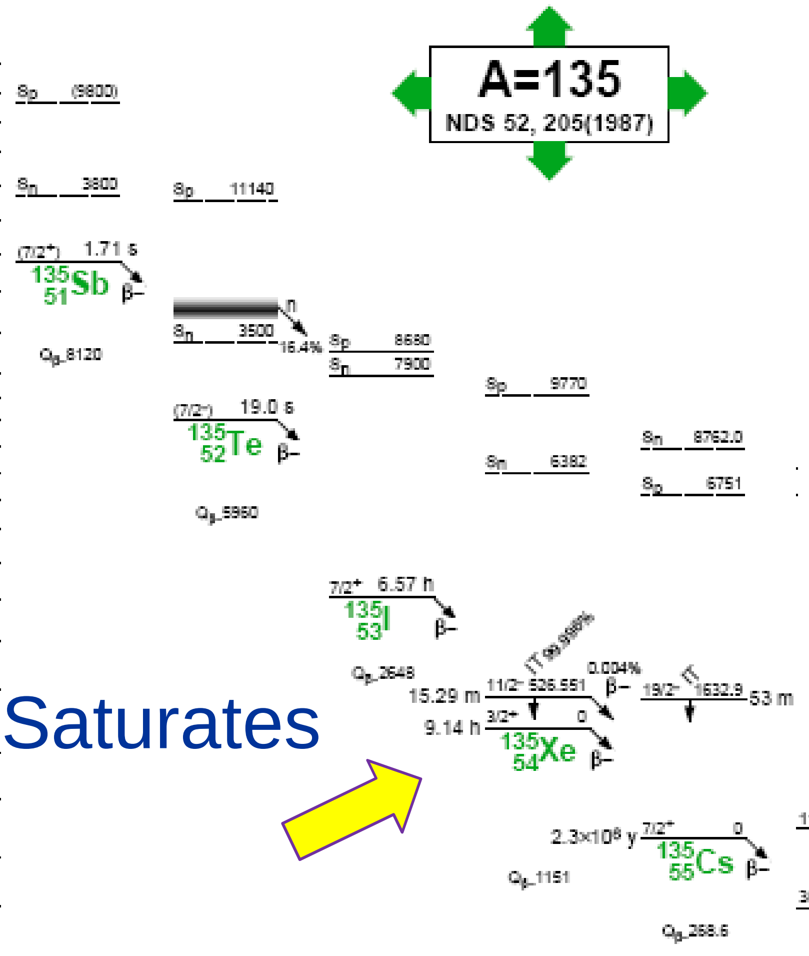
Work out the **multi-group to two-group** collapsing and investigate criticality.
Write down the appropriate matrices.

Poisoning

^{135}Xe

10^6 b



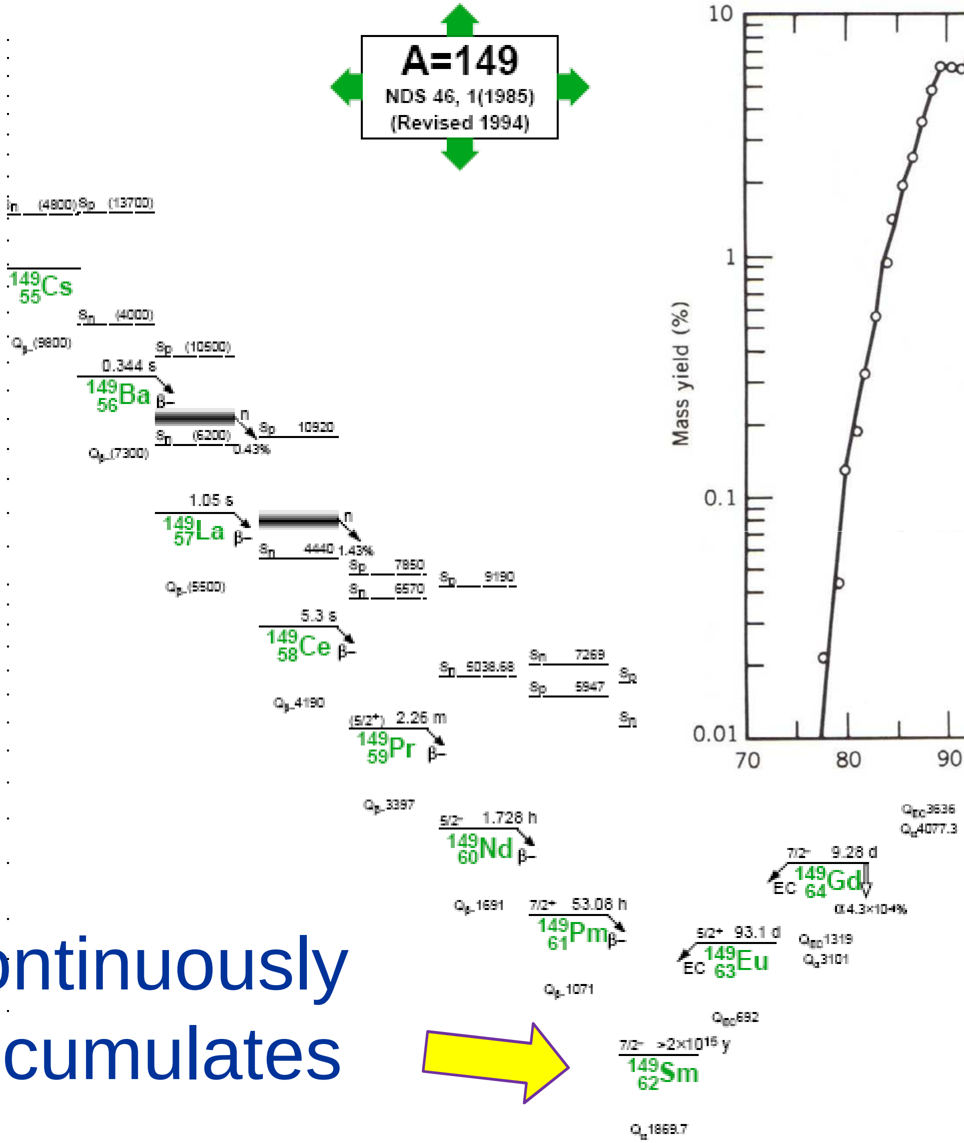


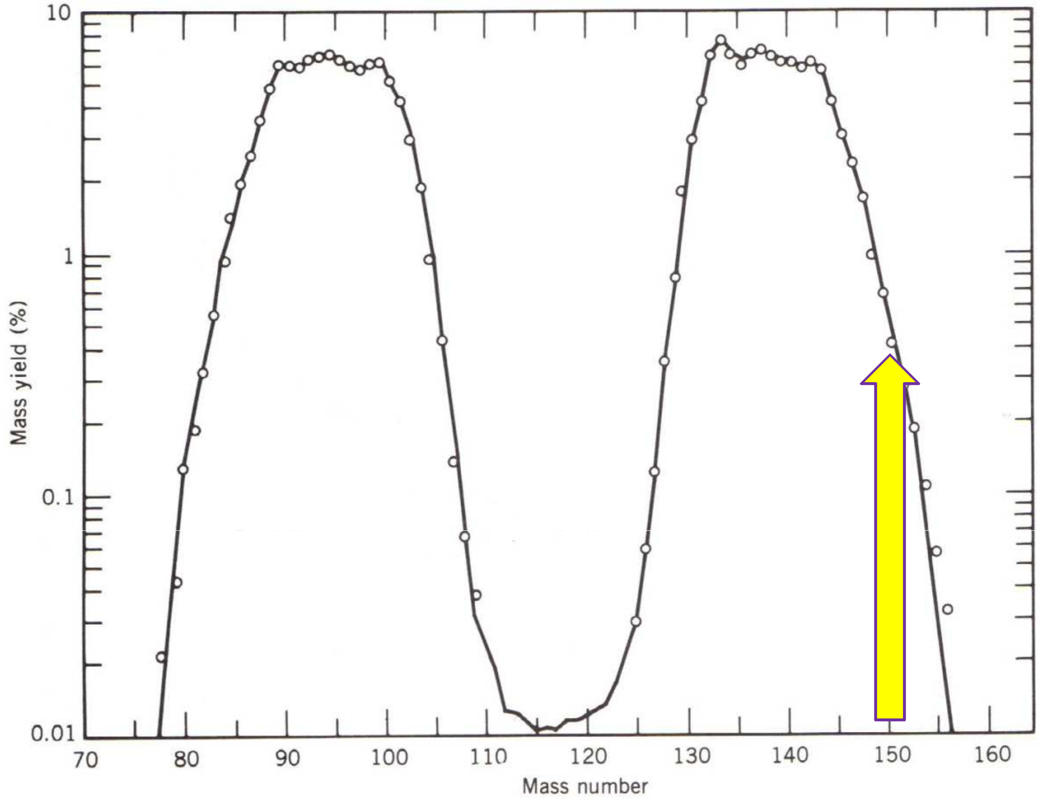
Saturates

Poisoning

^{149}Sm

10^5 b





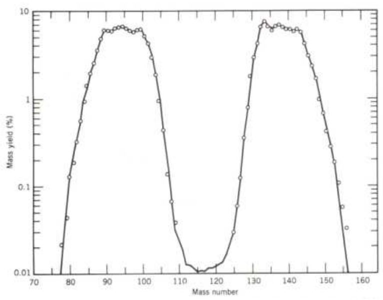
Continuously accumulates

Evaluator: B. Singh

Poisoning

- Not anticipated! Reactor shut down!

Time scale: Hours and days.

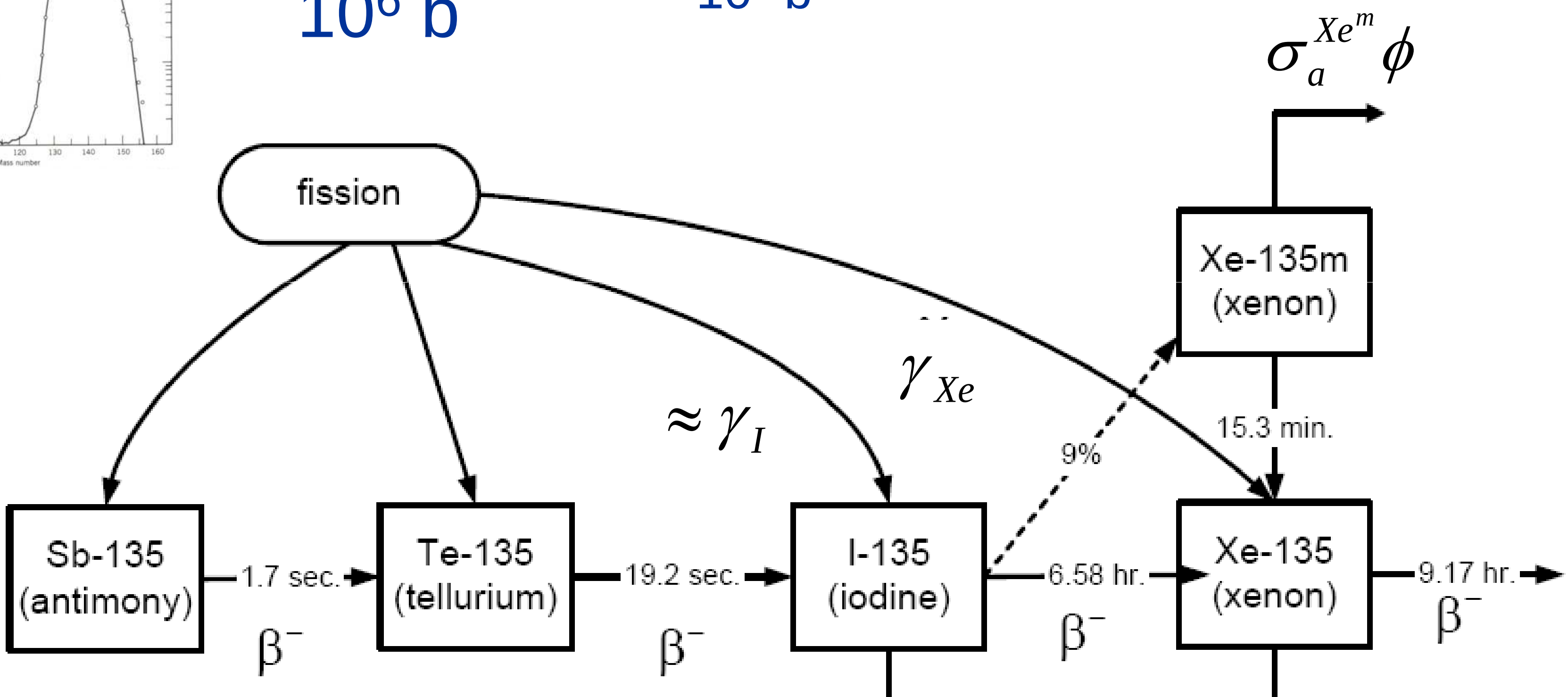


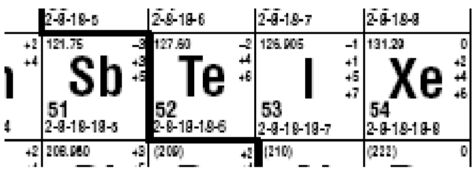
^{135}Xe

10^6 b

^{149}Sm

10^5 b





Poisoning

HW 28 Reactivity $\equiv \rho = \frac{k-1}{k}$, let us use k_∞ (Infinite reactor).

$$f_1 = \frac{\Sigma_a^{fuel}}{\Sigma_a^{fuel} + \Sigma_a^{clad} + \Sigma_a^{moderator}} \qquad \text{(critical)}$$

$$f_2 = \frac{\Sigma_a^{fuel}}{\Sigma_a^{fuel} + \Sigma_a^{clad} + \Sigma_a^{moderator} + \Sigma_a^{poison}}$$

Show that $\Delta\rho = \rho_2 - \rho_1 = -\frac{\Sigma_a^{poison}}{\Sigma_a^{fuel} + \Sigma_a^{clad} + \Sigma_a^{moderator}}$

Negative reactivity due to poison buildup. It is proportional to the amount of poison.

Poisoning

small

$$\frac{\partial I(\vec{r},t)}{\partial t} = \gamma_I \Sigma_f \phi(\vec{r},t) - \lambda_I I(\vec{r},t) - \sigma_a^I I(\vec{r},t)\phi(\vec{r},t)$$

$$\frac{\partial Xe(\vec{r},t)}{\partial t} = \gamma_{Xe} \Sigma_f \phi(\vec{r},t) + \lambda_I I(\vec{r},t) - \lambda_{Xe} Xe(\vec{r},t) - \sigma_a^{Xe} Xe(\vec{r},t)\phi(\vec{r},t)$$

Initial conditions?

- **Clean Core Startup.**
- Shutdown (later).

Assume no spacial dependence.

$I(0) = Xe(0) = 0$ Fresh Fuel.

and let us assume $\phi(t) = \phi(0) = \text{constant}.$

Poisoning

HW 29 Show that:

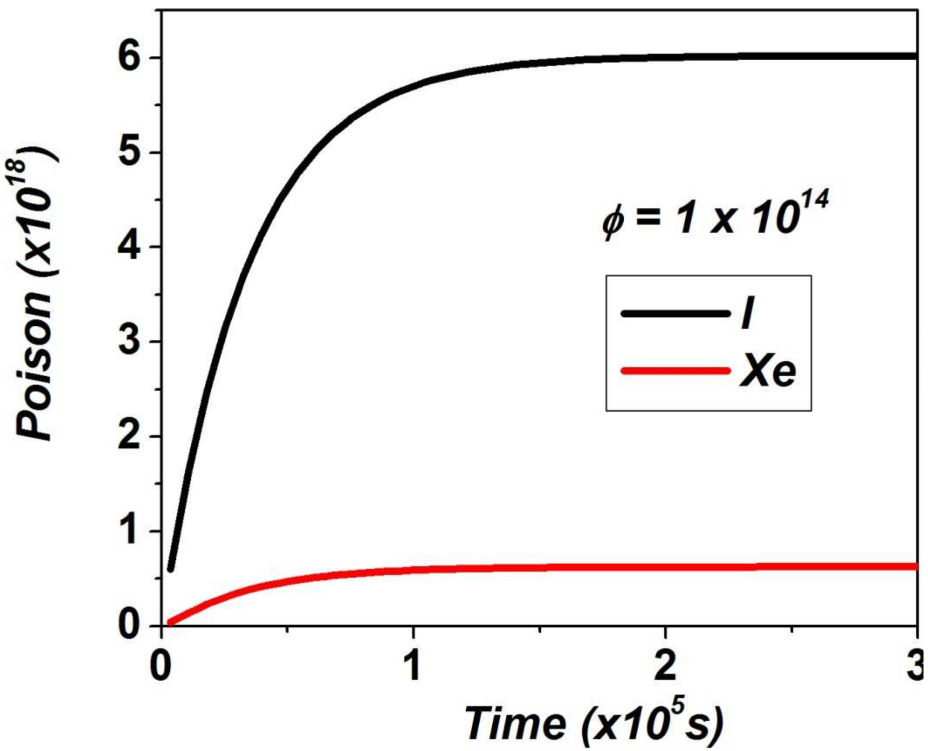
$$I(t)=\underbrace{\frac{\gamma_I \Sigma_f \phi_0}{\lambda_I}}_{I(\infty)}(1-e^{-\lambda_I t})$$

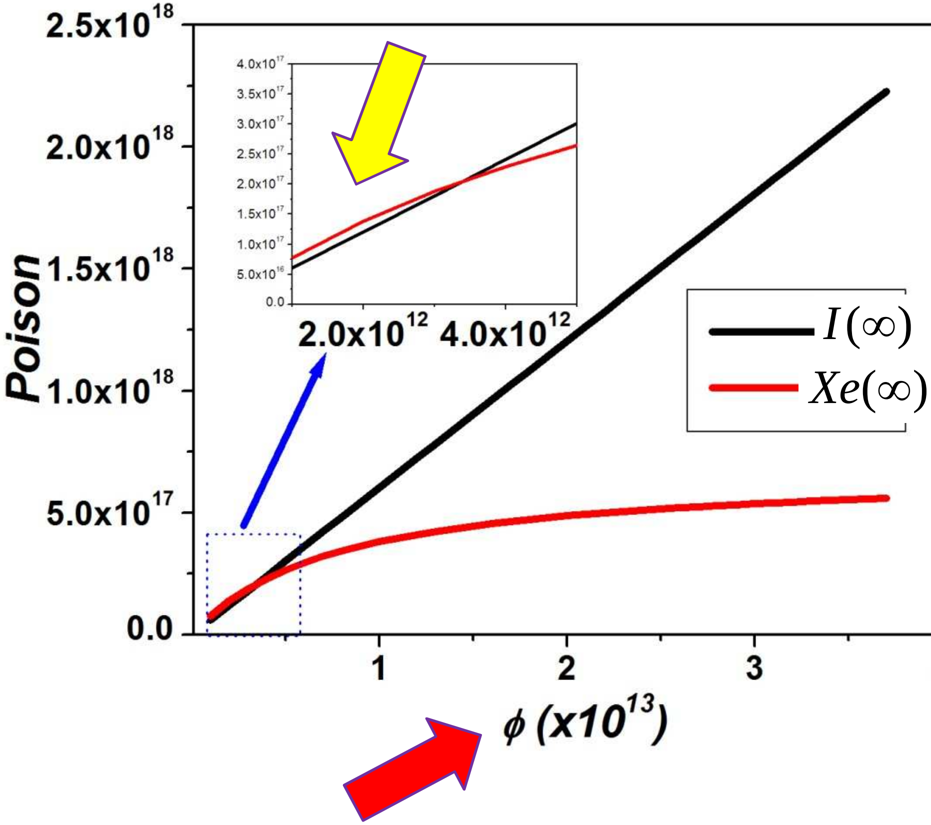
and

$$Xe(t)=\overbrace{\frac{(\gamma_I+\gamma_{Xe})\Sigma_f \phi_0}{\lambda_{Xe}+\sigma_a^{Xe}\phi_0}}^{Xe(\infty)}(1-e^{-(\lambda_{Xe}+\sigma_a^{Xe}\phi_0)t})$$

$$+\frac{\gamma_I \Sigma_f \phi_0}{\lambda_{Xe}-\lambda_I+\sigma_a^{Xe}\phi_0}(e^{-(\lambda_{Xe}+\sigma_a^{Xe}\phi_0)t}-e^{-\lambda_I t})$$

Poisoning





- Now, we know *Xe(t)*

$$\Delta\rho = -\frac{\sum_a^{poison}(t)}{\sum_a^{fuel}+\sum_a^{clad}+\sum_a^{moderator}} = -\frac{\sigma_a^{Xe}\,Xe(t)}{\sum_a^{fuel}+\sum_a^{clad}+\sum_a^{moderator}}$$

Poisoning

- **Shutdown.** After the reactor has been operating for a "long" time.

$$\frac{\partial I(\vec{r},t)}{\partial t} = \gamma_I \Sigma_f \phi(\vec{r},t) - \lambda_I I(\vec{r},t) - \sigma_a^I I(\vec{r},t)\phi(\vec{r},t)$$

$$\frac{\partial Xe(\vec{r},t)}{\partial t} = \gamma_{Xe} \Sigma_f \phi(\vec{r},t) + \lambda_I I(\vec{r},t) - \lambda_{Xe} Xe(\vec{r},t) - \sigma_a^{Xe} Xe(\vec{r},t)\phi(\vec{r},t)$$

$$I(0) = I(\infty)$$

$$Xe(0) = Xe(\infty)$$

$$\phi(t) = \phi(0) = 0.$$

$$\frac{\partial I(\vec{r},t)}{\partial t} = -\lambda_I I(\vec{r},t)$$

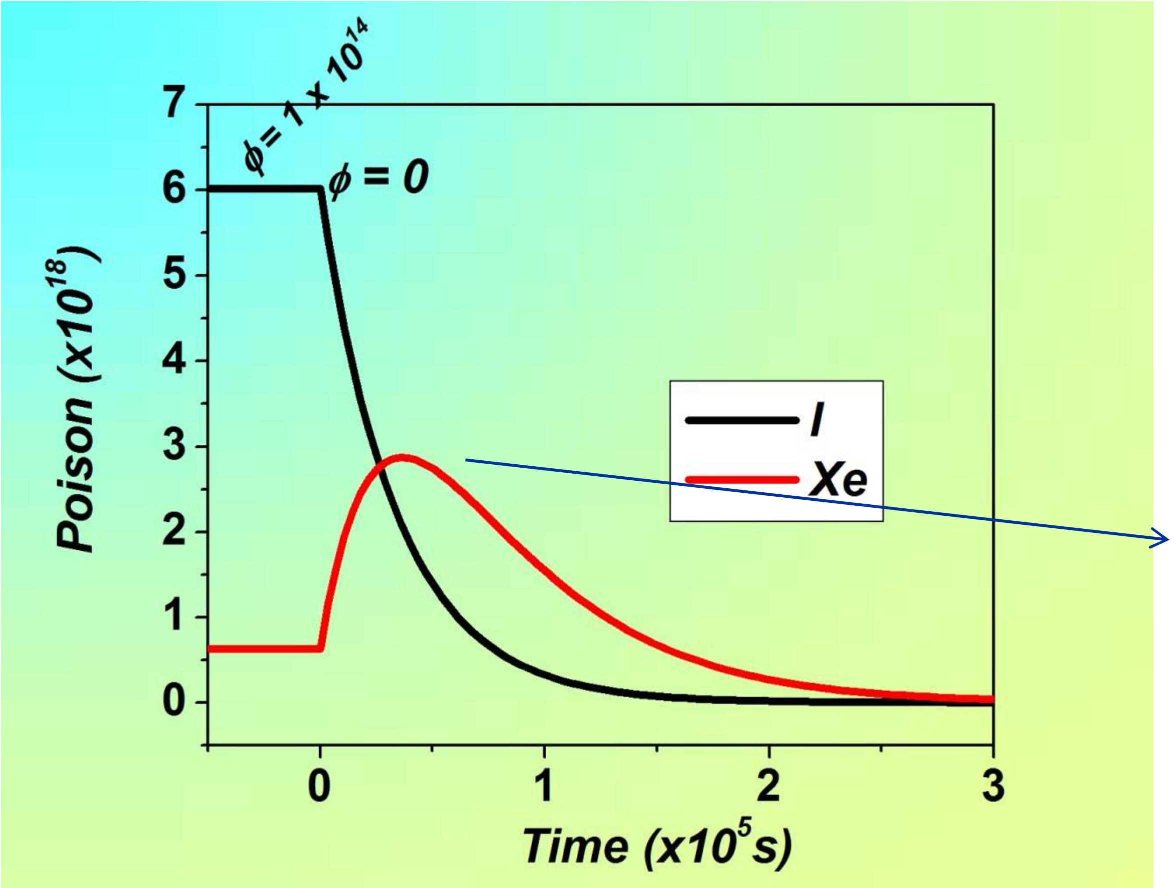
$$\frac{\partial Xe(\vec{r},t)}{\partial t} = \lambda_I I(\vec{r},t) - \lambda_{Xe} Xe(\vec{r},t)$$

Poisoning

HW 30 Show that $I(t) = I(\infty)e^{-\lambda_I t}$

$$Xe(t) = Xe(\infty)e^{-\lambda_{Xe}t} + \frac{\lambda_I I(\infty)}{\lambda_I - \lambda_{Xe}}(e^{-\lambda_{Xe}t} - e^{-\lambda_I t})$$

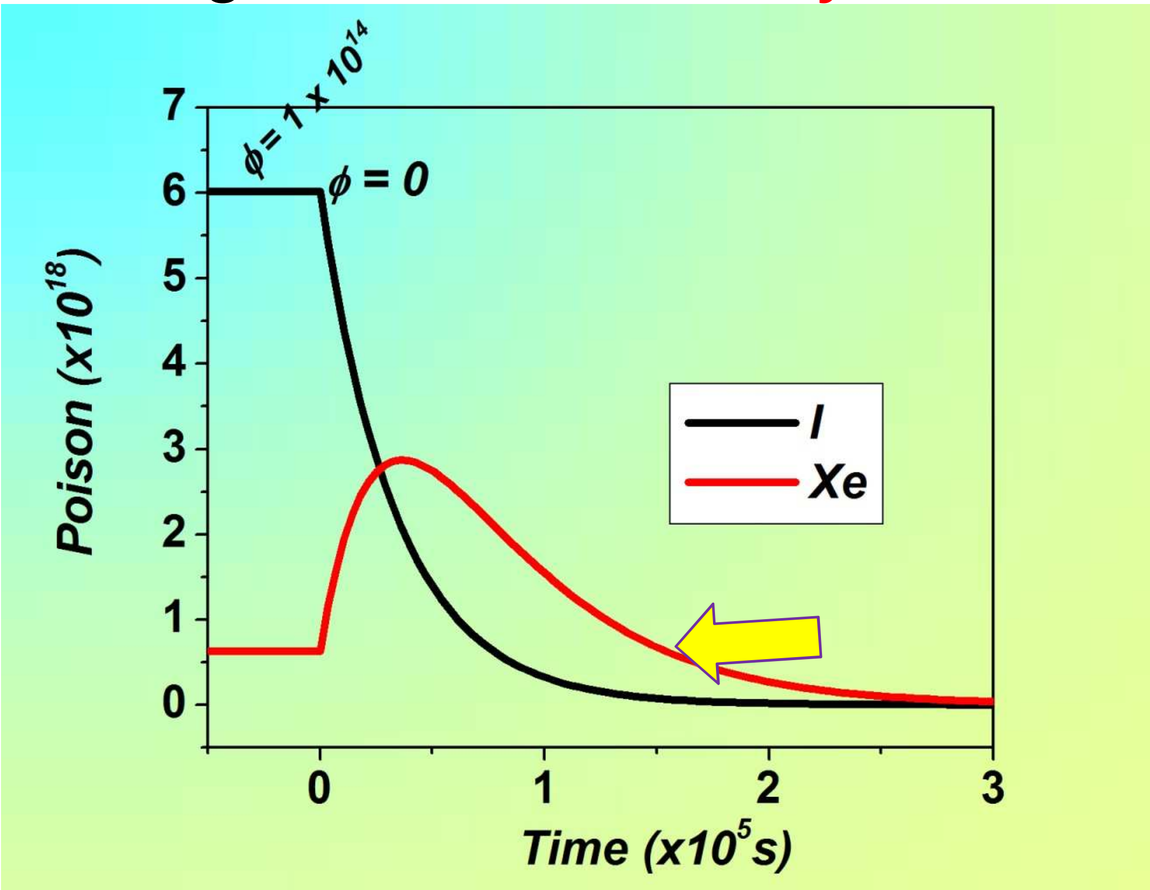
> 0 ?



Height of the peak depends on $I(\infty)$ and $Xe(\infty)$, *i.e.* depends on ϕ.

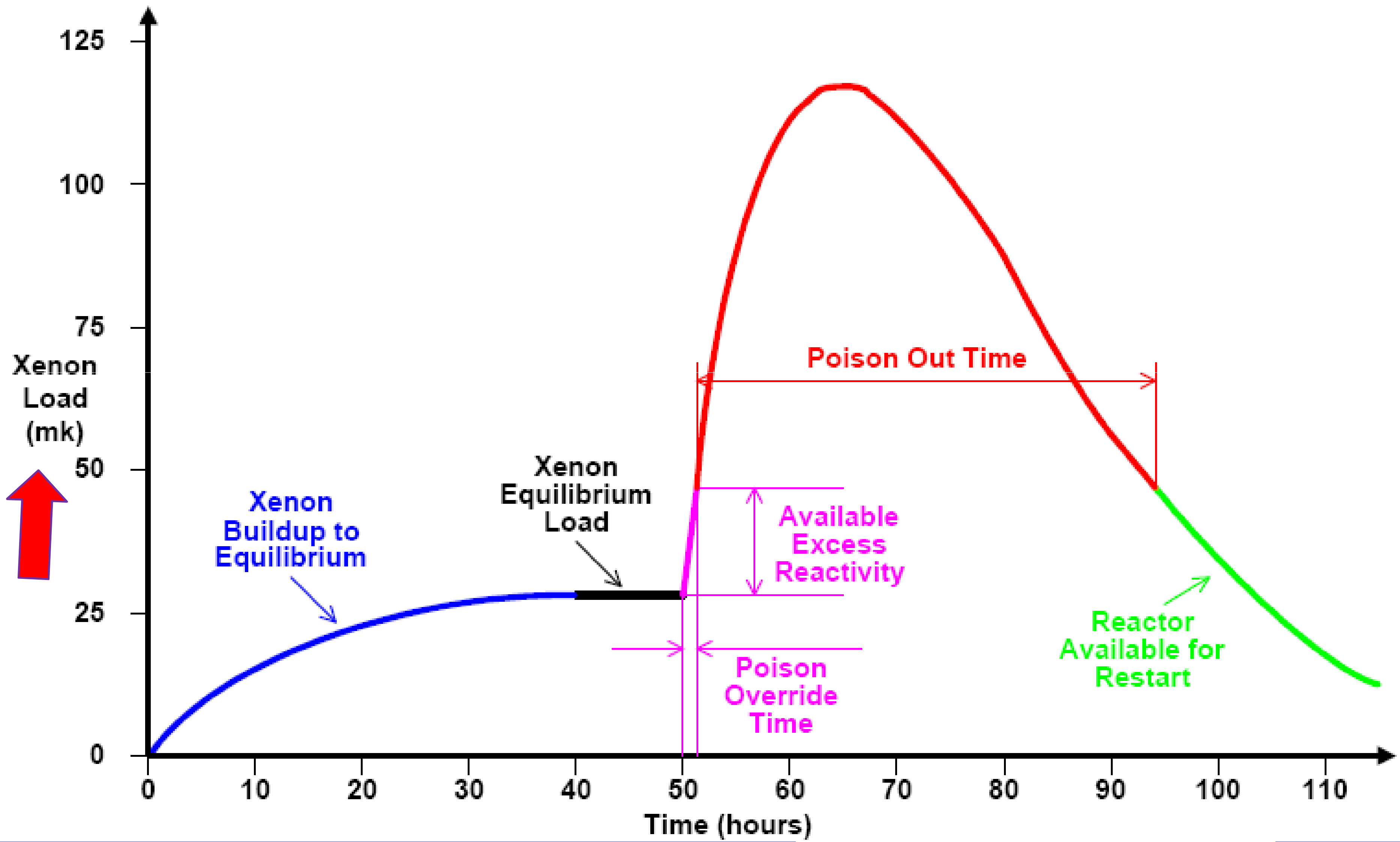
Poisoning

Shutdown ▶ Xe ↑ ▶ negative $\Delta\rho$ ▶ try to add positive reactivity ▶ move control rods out ▶ need to have enough reserve ▶ costly to do that.



If, the available excess reactivity can compensate for less than 30 minutes of poison buildup, can't startup again after ~30 minutes of shutdown, because you can't achieve criticality. Need to wait some 40 hours (in this case) for Xe to decay down.

Poisoning



125
100
75
50
25
0
Xenon
Load
(mk)
0
10
20
30
40
50
60
70
80
90
100
110
Time (hours)
Xenon
Buildup to
Equilibrium
Xenon
Equilibrium
Load
Poison Out Time
Available
Excess
Reactivity
Poison
Override
Time
Reactor
Available for
Restart

Poisoning

Strategies

- If you plan to shut down for "short maintenance", think about stepback.
- Examine different scenarios using this code from http://www.nuceng.ca/
- Prepare your own report, code, calculations, graphs, comments, conclusions etc…..
- Be creative and use whatever experience you gained during your study in this program.

20% (8 marks) of the final exam.

Poisoning

Xe Oscillations

• $\phi(\mathbf{r},t)$ (spacial dependence) ▶ flux ↑ locally ▶ Xe burnup ↑ ▶ ρ (reactivity) ↑ ▶ flux further ↑ ▶ control rods globally in ▶ flux ↓ elsewhere ▶ Xe burnup ↓ ▶ ρ ↓ ….. Xe oscillation but limited by opposite effect due to increase (decrease) of I in the high (low) flux region.

• In large reactors (compared to neutron diffusion length) local flux, power and temperature could reach unacceptable values for certain materials ▶ safety issues.

• Think of one sensor and one control rod ▶ feel average flux ▶ apparently OK ▶ more sensors and control rods to locate and deal with local changes.

Poisoning

Permanent Poisons

- ^{149}Sm has sizeable but lower cross section than ^{135}Xe.
- It does not decay.

$$\frac{\partial Sm(\vec{r},t)}{\partial t} \approx \gamma_{Sm} \Sigma_f(\vec{r},t)\phi(\vec{r},t) \ldots\ldots\ldots\ldots\ldots\ldots$$

????

- Accumulates with time.
- Consequences?????????