

Multi-group Model

$$\frac{1}{v_g} \frac{\partial}{\partial t} \phi_g(\vec{r}, t) = \chi_g \sum_{g'=1}^G v_{g'} \sum_{fg'}(\vec{r}) \phi_{g'}(\vec{r}, t) + \sum_{g'=1}^G \sum_{sg'g}(\vec{r}) \phi_{g'}(\vec{r}, t) + S_g^{ext} \\ - \sum_{ag}(\vec{r}) \phi_g(\vec{r}, t) - \sum_{sg}(\vec{r}) \phi_g(\vec{r}, t) + \vec{\nabla} \cdot D_g(\vec{r}) \vec{\nabla} \phi_g(\vec{r}, t)$$

Calculate group-averaged:

$$\frac{1}{v_g}, \sum_{fg'}(\vec{r}), \sum_{sg'g}(\vec{r}), \sum_{ag}(\vec{r}), \sum_{sg}(\vec{r}), D_g(\vec{r})$$

Or for,

$$\frac{1}{v_g} \frac{\partial}{\partial t} \phi_g(\vec{r}, t) = \chi_g \sum_{g'=1}^G v_{g'} \sum_{fg'}(\vec{r}) \phi_{g'}(\vec{r}, t) + \sum_{g'=1}^G \sum_{sg'g}(\vec{r}) \phi_{g'}(\vec{r}, t) + S_g^{ext} \\ - \sum_{rg}(\vec{r}) \phi_g(\vec{r}, t) - \sum_{sgg}(\vec{r}) \phi_g(\vec{r}, t) + \vec{\nabla} \cdot D_g(\vec{r}) \vec{\nabla} \phi_g(\vec{r}, t)$$

we need group-averaged $\sum_{rg}(\vec{r}), \sum_{sgg}(\vec{r})$

Multi-group Model

- *Group-averaged parameters?*
- *ENDF.*

$$\frac{1}{\nu(E)} \frac{\partial \phi(\vec{r}, E, t)}{\partial t} = \chi(E) \int_0^{\infty} \nu(E') \Sigma_f(\vec{r}, E') \phi(\vec{r}, E', t) dE'$$

Units!

$$+ \int_0^{\infty} \Sigma_s(\vec{r}, E' \rightarrow E) \phi(\vec{r}, E', t) dE' + S^{ext}$$

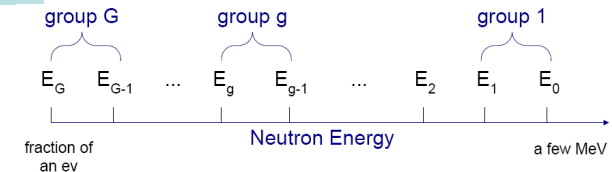
$$- \Sigma_a(\vec{r}, E) \phi(\vec{r}, E, t) - \Sigma_s(\vec{r}, E) \phi(\vec{r}, E, t)$$

$$+ \vec{\nabla} \cdot D(\vec{r}, E) \vec{\nabla} \phi(\vec{r}, E, t)$$

- *Integrate term by term over groups and equate to equation of multi-group model.*

Multi-group Model

- Define group flux $\phi_g(\vec{r}, t) \equiv \int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t) dE$



$$\frac{1}{v_g} \frac{\partial}{\partial t} \phi_g(\vec{r}, t) = \frac{\partial}{\partial t} \frac{\phi_g(\vec{r}, t)}{v_g} = \frac{\partial}{\partial t} \int_{E_g}^{E_{g-1}} \frac{1}{v(E)} \phi(\vec{r}, E, t) dE$$



$$\frac{1}{v_g} = \frac{\int_{E_g}^{E_{g-1}} \frac{1}{v(E)} \phi(\vec{r}, E, t) dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t) dE}$$

Multi-group Model

$$\vec{\nabla} \cdot D_g(\vec{r}) \vec{\nabla} \phi_g(\vec{r}, t) = \vec{\nabla} \cdot \int_{E_g}^{E_{g-1}} D(\vec{r}, E) \vec{\nabla} \phi(\vec{r}, E, t) dE$$



$$D_g(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} D(\vec{r}, E) \vec{\nabla} \phi(\vec{r}, E, t) dE}{\int_{E_g}^{E_{g-1}} \vec{\nabla} \phi(\vec{r}, E, t) dE}$$

Multi-group Model

$$\sum_{ag}(\vec{r})\phi_g(\vec{r}, t) = \int_{E_g}^{E_{g-1}} \sum_a(\vec{r}, E)\phi(\vec{r}, E, t)dE$$



$$\sum_{ag}(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} \sum_a(\vec{r}, E)\phi(\vec{r}, E, t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t)dE}$$

Multi-group Model

$$\sum_{sg}(\vec{r})\phi_g(\vec{r}, t) = \int_{E_g}^{E_{g-1}} \sum_s(\vec{r}, E)\phi(\vec{r}, E, t)dE$$



$$\sum_{sg}(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} \sum_s(\vec{r}, E)\phi(\vec{r}, E, t)dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t)dE}$$

Multi-group Model

$$\begin{aligned}
 \sum_{g'=1}^G \sum_{sg'g}(\vec{r}) \phi_{g'}(\vec{r}, t) &= \int_{E_g}^{E_{g-1}} \int_0^{\infty} \sum_s(\vec{r}, E' \rightarrow E) \phi(\vec{r}, E', t) dE' dE \\
 &= \int_{E_g}^{E_{g-1}} \sum_{g'=1}^G \int_{E_{g'}}^{E_{g'-1}} \sum_s(\vec{r}, E' \rightarrow E) \phi(\vec{r}, E', t) dE' dE \\
 &= \sum_{g'=1}^G \int_{E_g}^{E_{g-1}} \int_{E_{g'}}^{E_{g'-1}} \sum_s(\vec{r}, E' \rightarrow E) \phi(\vec{r}, E', t) dE' dE
 \end{aligned}$$



$$\sum_{sg'g}(\vec{r}) = \frac{1}{\phi_{g'}(\vec{r}, t)} \int_{E_g}^{E_{g-1}} \int_{E_{g'}}^{E_{g'-1}} \sum_s(E' \rightarrow E) \phi(\vec{r}, E', t) dE' dE$$

Multi-group Model

$$\begin{aligned} \chi_g \sum_{g'=1}^G \nu_{g'} \Sigma_{fg'}(\vec{r}) \phi_{g'}(\vec{r}, t) &= \int_{E_g}^{E_{g-1}} \chi(E) \int_0^{\infty} \nu(E') \Sigma_f(E') \phi(\vec{r}, E', t) dE' dE \\ &= \int_{E_g}^{E_{g-1}} \chi(E) dE \int_0^{\infty} \nu(E') \Sigma_f(E') \phi(\vec{r}, E', t) dE' \end{aligned}$$

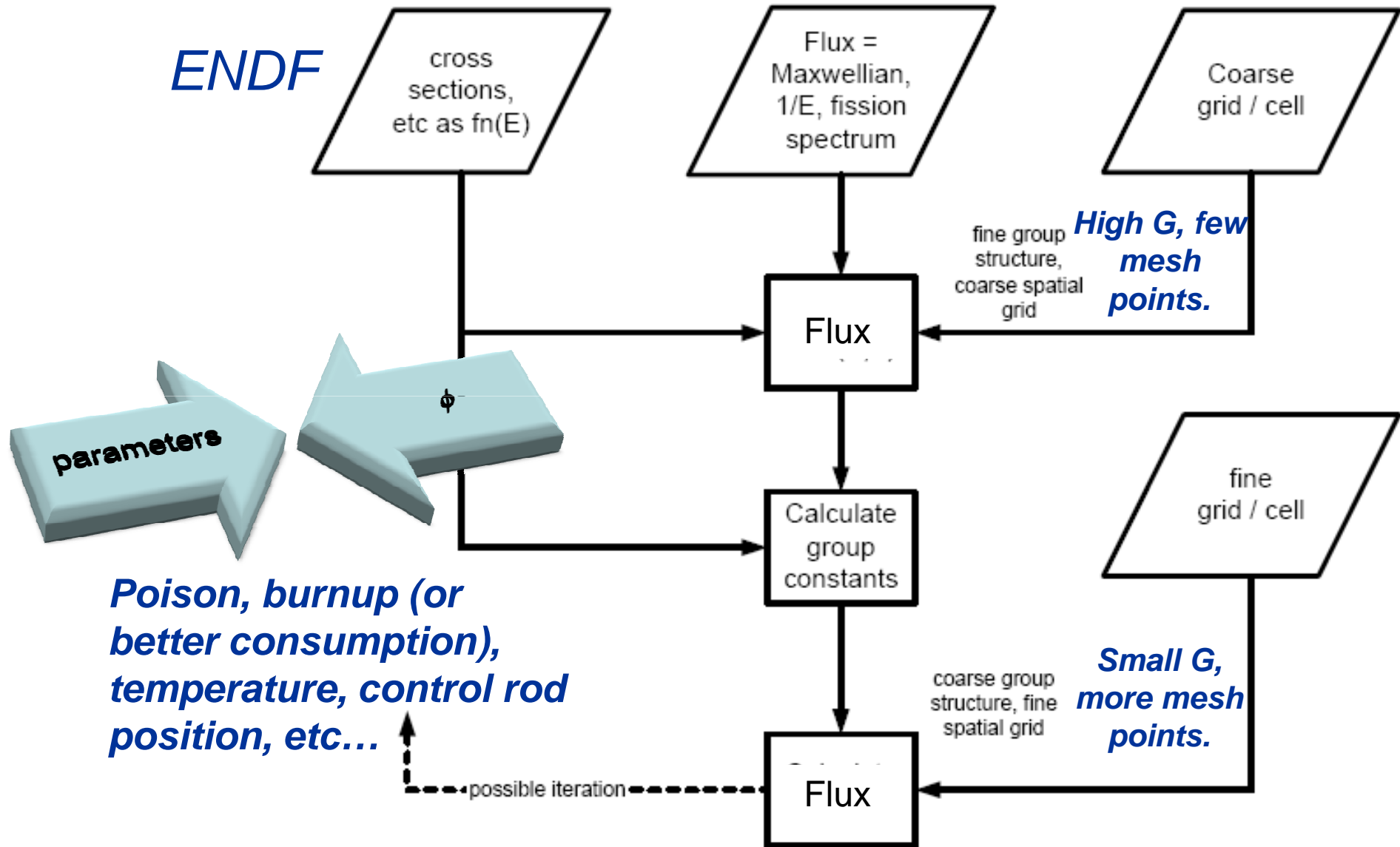
$$\chi_g = \int_{E_g}^{E_{g-1}} \chi(E) dE$$

$$\begin{aligned} &= \chi_g \int_0^{\infty} \nu(E') \Sigma_f(E') \phi(\vec{r}, E', t) dE' \\ &= \chi_g \sum_{g'=1}^G \int_{E_{g'}}^{E_{g'-1}} \nu(E') \Sigma_f(E') \phi(\vec{r}, E', t) dE' \end{aligned}$$



$$\nu_{g'} \Sigma_{fg'}(\vec{r}) = \frac{1}{\phi_{g'}(\vec{r}, t)} \int_{E_{g'}}^{E_{g'-1}} \nu(E') \Sigma_f(E') \phi(\vec{r}, E', t) dE'$$

Multi-group Model



Multi-group Model

What could make life a little easier?!

- No upscattering $\sum_{sg \setminus g} (\vec{r}) = 0$ for $g' > g$.
 set group G to include neutrons up to ~1 eV.

$$\sum_{g'=1}^G \sum_{sg \setminus g} (\vec{r}) \phi_{g'}(\vec{r}, t) \Rightarrow \sum_{g'=1}^{g-1} \sum_{sg \setminus g} (\vec{r}) \phi_{g'}(\vec{r}, t) + \underbrace{\left(\sum_{sgg} (\vec{r}) \phi_g(\vec{r}, t) \right)}$$

Your choice of how to tackle in-scattering.

- No group skipping when scattering down (directly coupled).

$$\sum_{g'=1}^G \sum_{sg \setminus g} (\vec{r}) \phi_{g'}(\vec{r}, t) \Rightarrow \sum_{s(g-1)g} (\vec{r}) \phi_{g-1}(\vec{r}, t) + \left(\sum_{sgg} (\vec{r}) \phi_g(\vec{r}, t) \right)$$

HW 27 How can we pledge this? What about H?

Multi-group Model

Criticality

Not all sources,
only fission.

$$M\phi = \frac{1}{k} F\phi$$

Not only sinks

Iterations.

$$\frac{1}{v_g} \frac{\partial}{\partial t} \phi_g(\vec{r}, t) = \chi_g \sum_{g'=1}^G v_{g'} \Sigma_{fg'}(\vec{r}) \phi_{g'}(\vec{r}, t) + \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg'}(\vec{r}) \phi_{g'}(\vec{r}, t) + S_g^{ext} - \Sigma_{rg}(\vec{r}) \phi_g(\vec{r}, t) + \vec{\nabla} \cdot D_g(\vec{r}) \vec{\nabla} \phi_g(\vec{r}, t)$$

No
upscatter

$$- \vec{\nabla} \cdot D_g(\vec{r}) \vec{\nabla} \phi_g(\vec{r}, t) + \Sigma_{rg}(\vec{r}) \phi_g(\vec{r}, t) - \sum_{\substack{g'=1 \\ g' \neq g}}^{g-1} \Sigma_{sg'}(\vec{r}) \phi_{g'}(\vec{r}, t)$$

Redundant when no upscatter.

$$= \frac{1}{K} \chi_g \sum_{g'=1}^G v_{g'} \Sigma_{fg'}(\vec{r}) \phi_{g'}(\vec{r}, t)$$

Multi-group Model

$$M = \begin{bmatrix} -\vec{\nabla} \cdot D_1 \vec{\nabla} + \Sigma_{r1} & 0 & 0 & \dots \\ -\Sigma_{s12} & -\vec{\nabla} \cdot D_2 \vec{\nabla} + \Sigma_{r2} & 0 & \dots \\ -\Sigma_{s13} & -\Sigma_{s23} & -\vec{\nabla} \cdot D_3 \vec{\nabla} + \Sigma_{r3} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

No upscatter

$$F = \begin{bmatrix} \chi_1 \nu_1 \Sigma_{f1} & \chi_1 \nu_2 \Sigma_{f2} & \chi_1 \nu_3 \Sigma_{f3} & \dots \\ \chi_2 \nu_1 \Sigma_{f1} & \chi_2 \nu_2 \Sigma_{f2} & \chi_2 \nu_3 \Sigma_{f3} & \dots \\ \chi_3 \nu_1 \Sigma_{f1} & \chi_3 \nu_2 \Sigma_{f2} & \chi_3 \nu_3 \Sigma_{f3} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \end{bmatrix}$$

Multi-group Model

$$M = \begin{bmatrix} -\vec{\nabla} \cdot D_1 \vec{\nabla} + \Sigma_{r1} & 0 & 0 & \dots \\ -\Sigma_{s12} & -\vec{\nabla} \cdot D_2 \vec{\nabla} + \Sigma_{r2} & 0 & \dots \\ 0 & -\Sigma_{s23} & -\vec{\nabla} \cdot D_3 \vec{\nabla} + \Sigma_{r3} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Directly coupled

No upscatter

$$F = \begin{bmatrix} \chi_1 \nu_1 \Sigma_{f1} & \chi_1 \nu_2 \Sigma_{f2} & \chi_1 \nu_3 \Sigma_{f3} & \dots \\ \chi_2 \nu_1 \Sigma_{f1} & \chi_2 \nu_2 \Sigma_{f2} & \chi_2 \nu_3 \Sigma_{f3} & \dots \\ \chi_3 \nu_1 \Sigma_{f1} & \chi_3 \nu_2 \Sigma_{f2} & \chi_3 \nu_3 \Sigma_{f3} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \end{bmatrix}$$

Multi-group Model

Multi-group \Rightarrow one-group

$$\phi_g(\vec{r}, t) \equiv \int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t) dE \Rightarrow \phi(\vec{r}, t) \equiv \int_0^{\infty} \phi(\vec{r}, E, t) dE$$

$$\frac{1}{v_g} = \frac{\int_{E_g}^{E_{g-1}} \frac{1}{v(E)} \phi(\vec{r}, E, t) dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t) dE} \Rightarrow \frac{1}{v} = \frac{\int_0^{\infty} \frac{1}{v(E)} \phi(\vec{r}, E, t) dE}{\int_0^{\infty} \phi(\vec{r}, E, t) dE}$$

Multi-group Model

$$D_g(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} D(\vec{r}, E) \vec{\nabla} \phi(\vec{r}, E, t) dE}{\int_{E_g}^{E_{g-1}} \vec{\nabla} \phi(\vec{r}, E, t) dE} \Rightarrow D(\vec{r}) = \frac{\int_0^{\infty} D(\vec{r}, E) \vec{\nabla} \phi(\vec{r}, E, t) dE}{\int_0^{\infty} \vec{\nabla} \phi(\vec{r}, E, t) dE}$$

$$\sum_{ag}(\vec{r}) = \frac{\int_{E_g}^{E_{g-1}} \sum_a(\vec{r}, E) \phi(\vec{r}, E, t) dE}{\int_{E_g}^{E_{g-1}} \phi(\vec{r}, E, t) dE} \Rightarrow \sum_a(\vec{r}) = \frac{\int_0^{\infty} \sum_a(\vec{r}, E) \phi(\vec{r}, E, t) dE}{\int_0^{\infty} \phi(\vec{r}, E, t) dE}$$

Multi-group Model

$$\sum_{g=1}^G \sum_{sg} (\vec{r}) \phi_{g'}(\vec{r}, t) - \sum_{sg} (\vec{r}) \phi_g(\vec{r}, t) \Rightarrow 0 \text{ when } G = 1$$

$$\chi_g = \int_{E_g}^{E_{g-1}} \chi(E) dE \Rightarrow \chi = \int_0^{\infty} \chi(E) dE = 1$$

$$\sum_{g=1}^G \nu_{g'} \sum_{fg'} (\vec{r}) \phi_{g'}(\vec{r}, t) \Rightarrow \nu \sum_f (\vec{r}) \phi(\vec{r}, t) \text{ when } G = 1$$

Multi-group Model

Substituting all of the above into

$$\begin{aligned} \frac{1}{v_g} \frac{\partial}{\partial t} \phi_g(\vec{r}, t) = & \chi_g \sum_{g'=1}^G \nu_{g'} \sum_{fg'}(\vec{r}) \phi_{g'}(\vec{r}, t) + \sum_{g'=1}^G \sum_{sg'g}(\vec{r}) \phi_{g'}(\vec{r}, t) + S_g^{ext} \\ & - \sum_{ag}(\vec{r}) \phi_g(\vec{r}, t) - \sum_{sg}(\vec{r}) \phi_g(\vec{r}, t) + \vec{\nabla} \cdot D_g(\vec{r}) \vec{\nabla} \phi_g(\vec{r}, t) \end{aligned}$$

yields

$$\begin{aligned} \frac{1}{v} \frac{\partial}{\partial t} \phi(\vec{r}, t) = & \nu \sum_f(\vec{r}) \phi(\vec{r}, t) + S^{ext} \\ & - \sum_a(\vec{r}) \phi(\vec{r}, t) + \vec{\nabla} \cdot D(\vec{r}) \vec{\nabla} \phi(\vec{r}, t) \end{aligned}$$

which is the one-group diffusion equation.

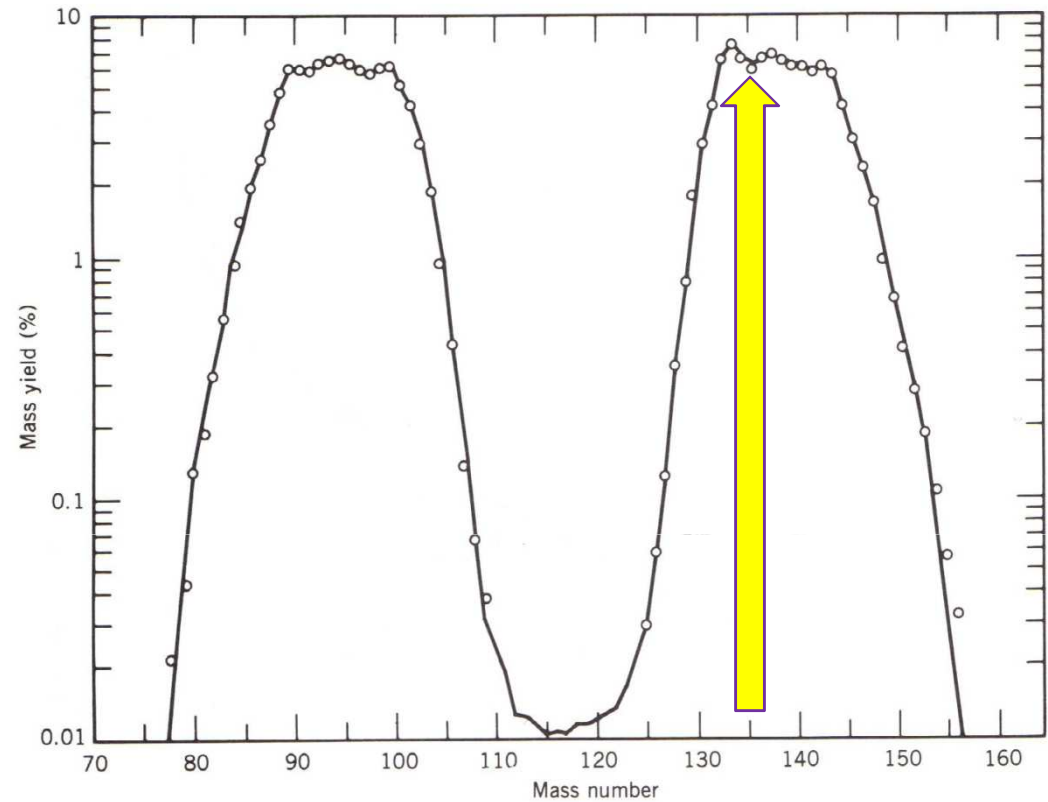
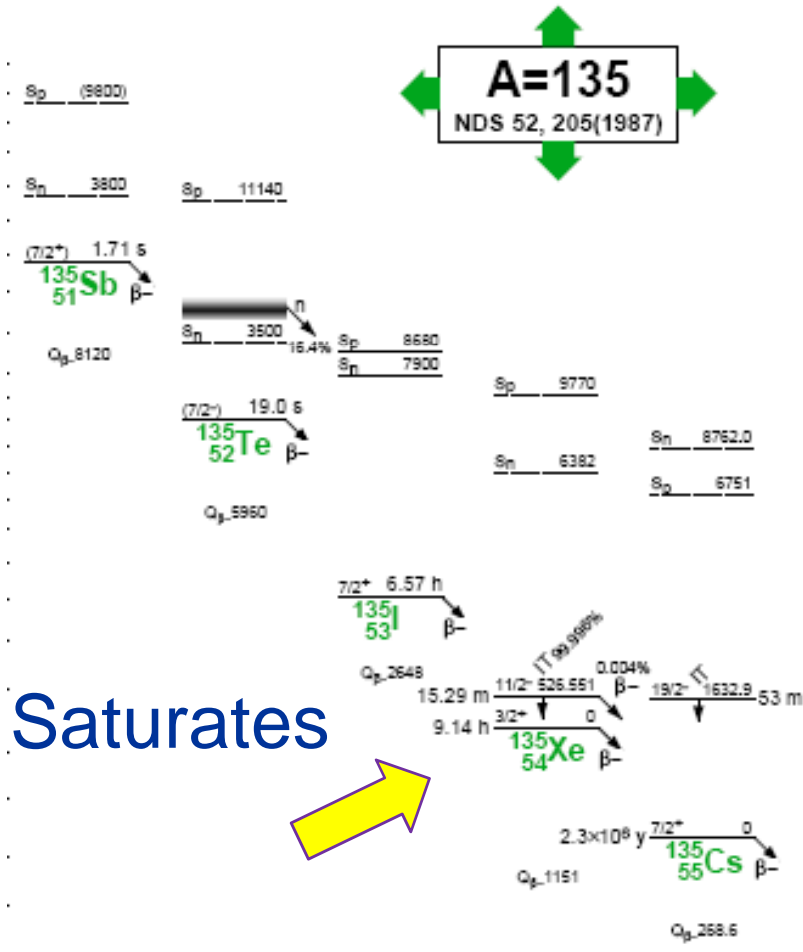
Multi-group Model

Project 3

Work out the **multi-group to two-group** collapsing and investigate criticality.
Write down the appropriate matrices.

Poisoning

^{135}Xe
 10^6 b



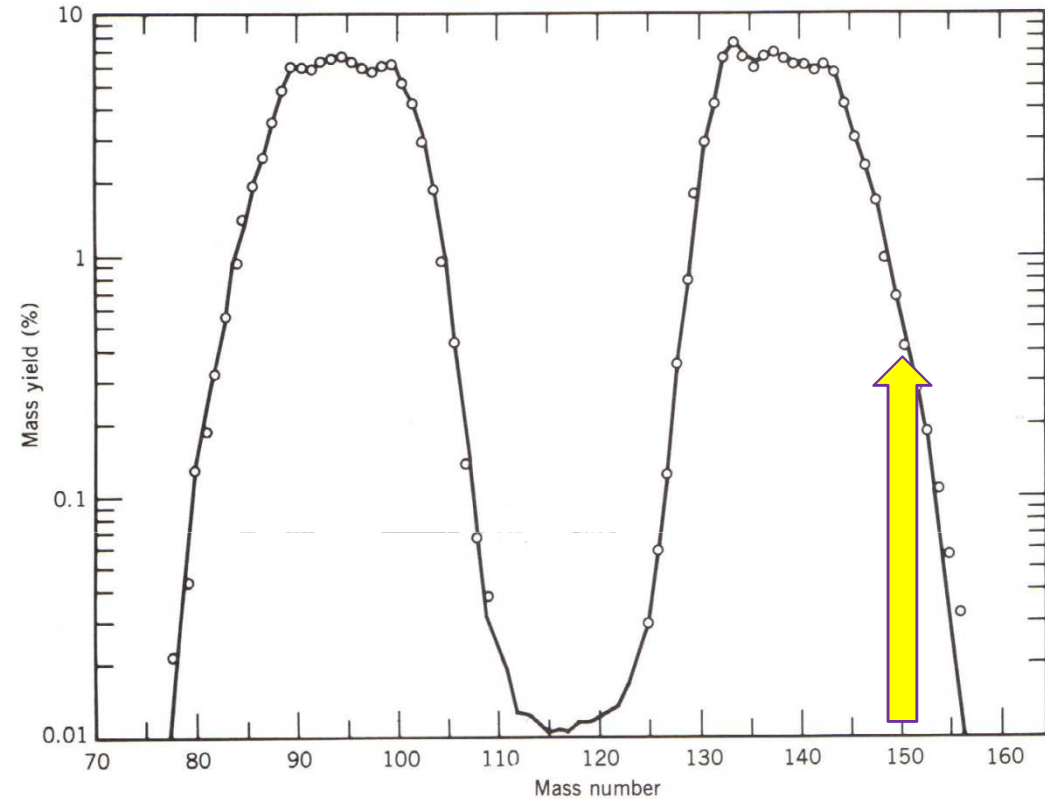
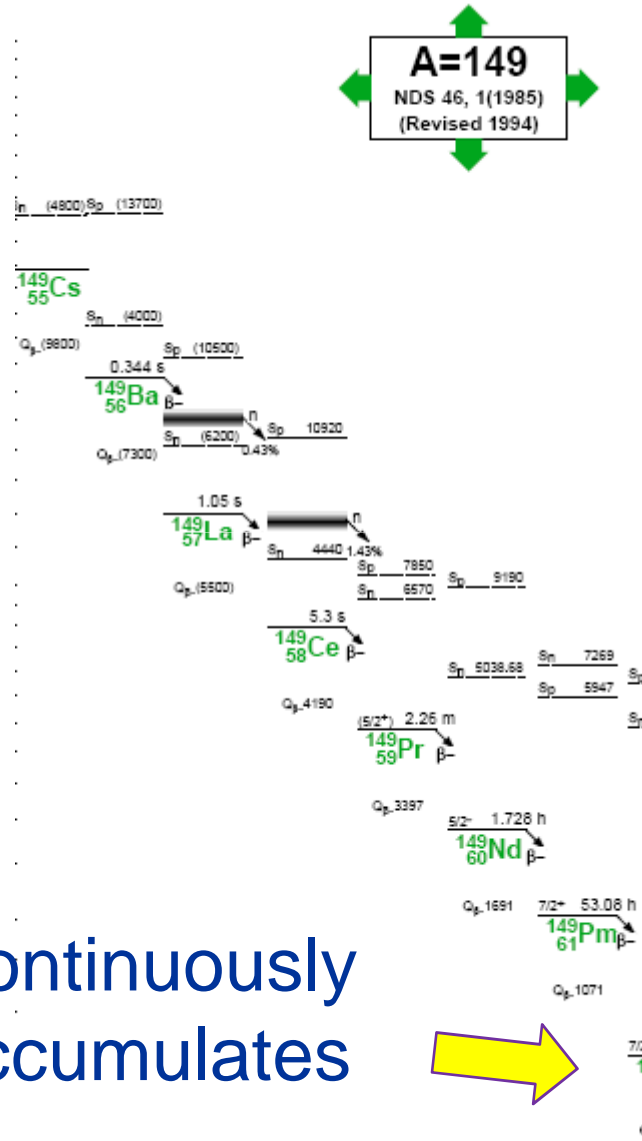
^{135}Ce ($1/2^+$, 17.7 h) EC

^{135}La ($5/2^+$, 19.5 h) EC

Evaluator: Yu.V. Sergeenkov

Poisoning

^{149}Sm
 10^5 b



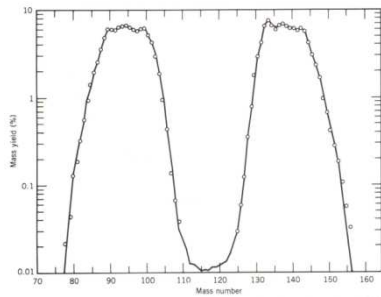
Continuously
accumulates

Evaluator: B. Singh

Poisoning

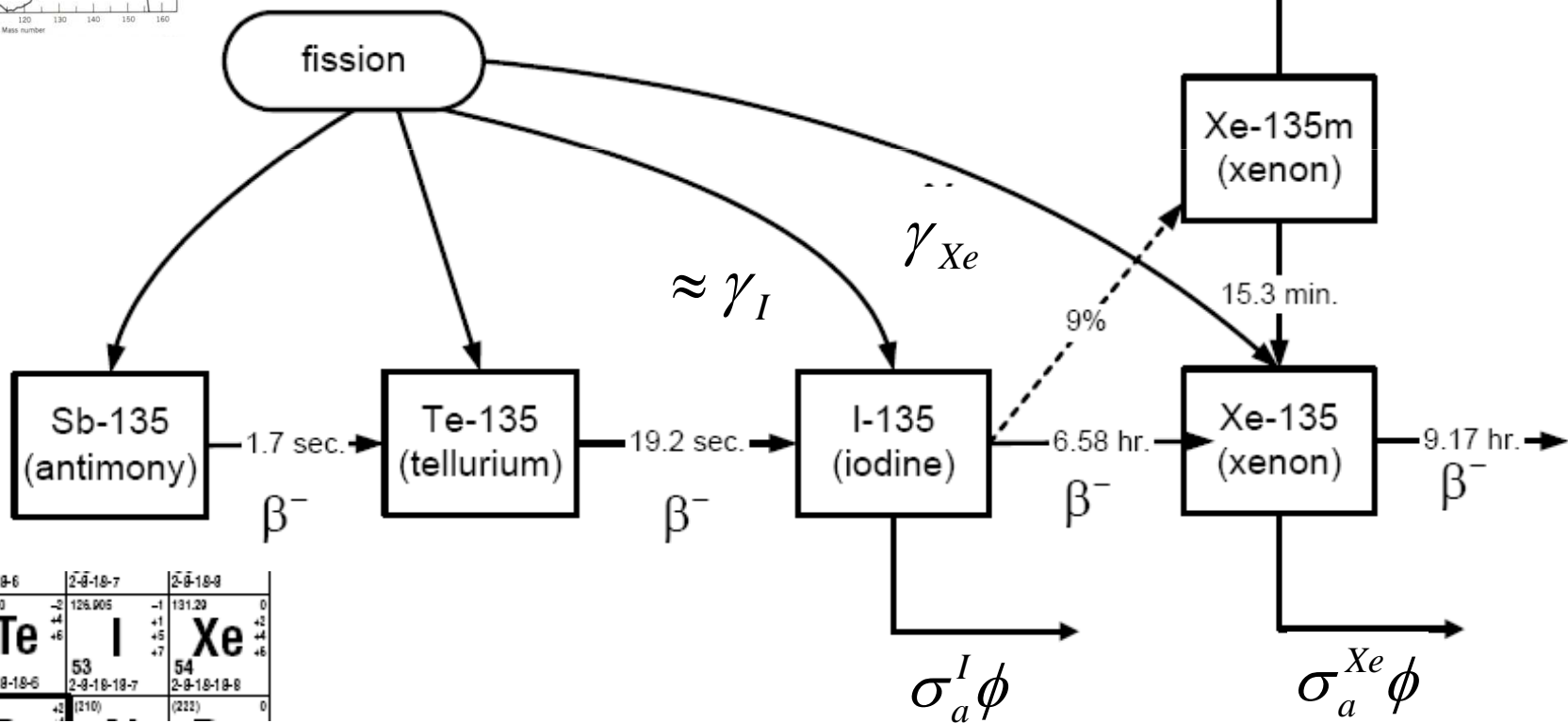
- Not anticipated! Reactor shut down!

Time scale:
Hours and days.



^{135}Xe
 10^6 b

^{149}Sm
 10^5 b

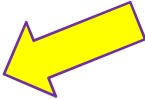


	^{135}Sb	^{135}Te	^{135}I	^{135}Xe
Half-life	$2.6 \cdot 10^{-5}$	$2.6 \cdot 10^{-6}$	$2.6 \cdot 10^{-7}$	$2.6 \cdot 10^{-8}$
λ	121.75	127.80	126.005	131.20
β^- energy (MeV)	+4	+3	+2	+1
β^- energy (MeV)	+5	+4	+3	+2
β^- energy (MeV)	+6	+5	+4	+3
β^- energy (MeV)	+7	+6	+5	+4
β^- energy (MeV)	+8	+7	+6	+5
β^- energy (MeV)	+9	+8	+7	+6
β^- energy (MeV)	+10	+9	+8	+7
β^- energy (MeV)	+11	+10	+9	+8
β^- energy (MeV)	+12	+11	+10	+9
β^- energy (MeV)	+13	+12	+11	+10
β^- energy (MeV)	+14	+13	+12	+11
β^- energy (MeV)	+15	+14	+13	+12
β^- energy (MeV)	+16	+15	+14	+13
β^- energy (MeV)	+17	+16	+15	+14
β^- energy (MeV)	+18	+17	+16	+15
β^- energy (MeV)	+19	+18	+17	+16
β^- energy (MeV)	+20	+19	+18	+17
β^- energy (MeV)	+21	+20	+19	+18
β^- energy (MeV)	+22	+21	+20	+19
β^- energy (MeV)	+23	+22	+21	+20
β^- energy (MeV)	+24	+23	+22	+21
β^- energy (MeV)	+25	+24	+23	+22
β^- energy (MeV)	+26	+25	+24	+23
β^- energy (MeV)	+27	+26	+25	+24
β^- energy (MeV)	+28	+27	+26	+25
β^- energy (MeV)	+29	+28	+27	+26
β^- energy (MeV)	+30	+29	+28	+27
β^- energy (MeV)	+31	+30	+29	+28
β^- energy (MeV)	+32	+31	+30	+29
β^- energy (MeV)	+33	+32	+31	+30
β^- energy (MeV)	+34	+33	+32	+31
β^- energy (MeV)	+35	+34	+33	+32
β^- energy (MeV)	+36	+35	+34	+33
β^- energy (MeV)	+37	+36	+35	+34
β^- energy (MeV)	+38	+37	+36	+35
β^- energy (MeV)	+39	+38	+37	+36
β^- energy (MeV)	+40	+39	+38	+37
β^- energy (MeV)	+41	+40	+39	+38
β^- energy (MeV)	+42	+41	+40	+39
β^- energy (MeV)	+43	+42	+41	+40
β^- energy (MeV)	+44	+43	+42	+41
β^- energy (MeV)	+45	+44	+43	+42
β^- energy (MeV)	+46	+45	+44	+43
β^- energy (MeV)	+47	+46	+45	+44
β^- energy (MeV)	+48	+47	+46	+45
β^- energy (MeV)	+49	+48	+47	+46
β^- energy (MeV)	+50	+49	+48	+47
β^- energy (MeV)	+51	+50	+49	+48
β^- energy (MeV)	+52	+51	+50	+49
β^- energy (MeV)	+53	+52	+51	+50
β^- energy (MeV)	+54	+53	+52	+51
β^- energy (MeV)	+55	+54	+53	+52
β^- energy (MeV)	+56	+55	+54	+53
β^- energy (MeV)	+57	+56	+55	+54
β^- energy (MeV)	+58	+57	+56	+55
β^- energy (MeV)	+59	+58	+57	+56
β^- energy (MeV)	+60	+59	+58	+57
β^- energy (MeV)	+61	+60	+59	+58
β^- energy (MeV)	+62	+61	+60	+59
β^- energy (MeV)	+63	+62	+61	+60
β^- energy (MeV)	+64	+63	+62	+61
β^- energy (MeV)	+65	+64	+63	+62
β^- energy (MeV)	+66	+65	+64	+63
β^- energy (MeV)	+67	+66	+65	+64
β^- energy (MeV)	+68	+67	+66	+65
β^- energy (MeV)	+69	+68	+67	+66
β^- energy (MeV)	+70	+69	+68	+67
β^- energy (MeV)	+71	+70	+69	+68
β^- energy (MeV)	+72	+71	+70	+69
β^- energy (MeV)	+73	+72	+71	+70
β^- energy (MeV)	+74	+73	+72	+71
β^- energy (MeV)	+75	+74	+73	+72
β^- energy (MeV)	+76	+75	+74	+73
β^- energy (MeV)	+77	+76	+75	+74
β^- energy (MeV)	+78	+77	+76	+75
β^- energy (MeV)	+79	+78	+77	+76
β^- energy (MeV)	+80	+79	+78	+77
β^- energy (MeV)	+81	+80	+79	+78
β^- energy (MeV)	+82	+81	+80	+79
β^- energy (MeV)	+83	+82	+81	+80
β^- energy (MeV)	+84	+83	+82	+81
β^- energy (MeV)	+85	+84	+83	+82
β^- energy (MeV)	+86	+85	+84	+83
β^- energy (MeV)	+87	+86	+85	+84
β^- energy (MeV)	+88	+87	+86	+85
β^- energy (MeV)	+89	+88	+87	+86
β^- energy (MeV)	+90	+89	+88	+87
β^- energy (MeV)	+91	+90	+89	+88
β^- energy (MeV)	+92	+91	+90	+89
β^- energy (MeV)	+93	+92	+91	+90
β^- energy (MeV)	+94	+93	+92	+91
β^- energy (MeV)	+95	+94	+93	+92
β^- energy (MeV)	+96	+95	+94	+93
β^- energy (MeV)	+97	+96	+95	+94
β^- energy (MeV)	+98	+97	+96	+95
β^- energy (MeV)	+99	+98	+97	+96
β^- energy (MeV)	+100	+99	+98	+97
β^- energy (MeV)	+101	+100	+99	+98
β^- energy (MeV)	+102	+101	+100	+99
β^- energy (MeV)	+103	+102	+101	+100
β^- energy (MeV)	+104	+103	+102	+101
β^- energy (MeV)	+105	+104	+103	+102
β^- energy (MeV)	+106	+105	+104	+103
β^- energy (MeV)	+107	+106	+105	+104
β^- energy (MeV)	+108	+107	+106	+105
β^- energy (MeV)	+109	+108	+107	+106
β^- energy (MeV)	+110	+109	+108	+107
β^- energy (MeV)	+111	+110	+109	+108
β^- energy (MeV)	+112	+111	+110	+109
β^- energy (MeV)	+113	+112	+111	+110
β^- energy (MeV)	+114	+113	+112	+111
β^- energy (MeV)	+115	+114	+113	+112
β^- energy (MeV)	+116	+115	+114	+113
β^- energy (MeV)	+117	+116	+115	+114
β^- energy (MeV)	+118	+117	+116	+115
β^- energy (MeV)	+119	+118	+117	+116
β^- energy (MeV)	+120	+119	+118	+117
β^- energy (MeV)	+121	+120	+119	+118
β^- energy (MeV)	+122	+121	+120	+119
β^- energy (MeV)	+123	+122	+121	+120
β^- energy (MeV)	+124	+123	+122	+121
β^- energy (MeV)	+125	+124	+123	+122
β^- energy (MeV)	+126	+125	+124	+123
β^- energy (MeV)	+127	+126	+125	+124
β^- energy (MeV)	+128	+127	+126	+125
β^- energy (MeV)	+129	+128	+127	+126
β^- energy (MeV)	+130	+129	+128	+127
β^- energy (MeV)	+131	+130	+129	+128
β^- energy (MeV)	+132	+131	+130	+129
β^- energy (MeV)	+133	+132	+131	+130
β^- energy (MeV)	+134	+133	+132	+131
β^- energy (MeV)	+135	+134	+133	+132
β^- energy (MeV)	+136	+135	+134	+133
β^- energy (MeV)	+137	+136	+135	+134
β^- energy (MeV)	+138	+137	+136	+135
β^- energy (MeV)	+139	+138	+137	+136
β^- energy (MeV)	+140	+139	+138	+137
β^- energy (MeV)	+141	+140	+139	+138
β^- energy (MeV)	+142	+141	+140	+139
β^- energy (MeV)	+143	+142	+141	+140
β^- energy (MeV)	+144	+143	+142	+141
β^- energy (MeV)	+145	+144	+143	+142
β^- energy (MeV)	+146	+145	+144	+143
β^- energy (MeV)	+147	+146	+145	+144
β^- energy (MeV)	+148	+147	+146	+145
β^- energy (MeV)	+149	+148	+147	+146
β^- energy (MeV)	+150	+149	+148	+147
β^- energy (MeV)	+151	+150	+149	+148
β^- energy (MeV)	+152	+151	+150	+149
β^- energy (MeV)	+153	+152	+151	+150
β^- energy (MeV)	+154	+153	+152	+151
β^- energy (MeV)	+155	+154	+153	+152
β^- energy (MeV)	+156	+155	+154	+153
β^- energy (MeV)	+157	+156	+155	+154
β^- energy (MeV)	+158	+157	+156	+155
β^- energy (MeV)	+159	+158	+157	+156
β^- energy (MeV)	+160	+159	+158	+157
β^- energy (MeV)	+161	+160	+159	+158
β^- energy (MeV)	+162	+161	+160	+159
β^- energy (MeV)	+163	+162	+161	+160
β^- energy (MeV)	+164	+163	+162	+161
β^- energy (MeV)	+165	+164	+163	+162
β^- energy (MeV)	+166	+165	+164	+163
β^- energy (MeV)	+167	+166	+165	+164
β^- energy (MeV)	+168	+167	+166	+165
β^- energy (MeV)	+169	+168	+167	+166
β^- energy (MeV)	+170	+169	+168	+167
β^- energy (MeV)	+171	+170	+169	+168
β^- energy (MeV)	+172	+171	+170	+169
β^- energy (MeV)	+173	+172	+171	+170
β^- energy (MeV)	+174	+173	+172	+171
β^- energy (MeV)	+175	+174	+173	+172
β^- energy (MeV)	+176	+175	+174	+173
β^- energy (MeV)	+177	+176	+175	+174
β^- energy (MeV)	+178	+177	+176	+175
β^- energy (MeV)	+179	+178	+177	+176
β^- energy (MeV)	+180	+179	+178	+177
β^- energy (MeV)	+181	+180	+179	+178
β^- energy (MeV)	+182	+181	+180	+179
β^- energy (MeV)	+183	+182	+181	+180
β^- energy (MeV)	+184	+183	+182	+181
β^- energy (MeV)	+185	+184	+183	+182
β^- energy (MeV)	+186	+185	+184	+183
β^- energy (MeV)	+187	+186	+185	+184
β^- energy (MeV)	+188	+187	+186	+185
β^- energy (MeV)	+189	+188	+187	+186
β^- energy (MeV)	+190	+189	+188	+187
β^- energy (MeV)	+191	+190	+189	+188
β^- energy (MeV)	+192	+191	+190	+189
β^- energy (MeV)	+193	+192	+191	+190
β^- energy (MeV)	+194	+193	+192	+191
β^- energy (MeV)	+195	+194	+193	+192
β^- energy (MeV)	+196	+195	+194	+193
β				

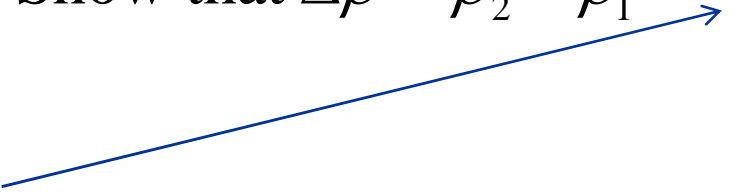
Poisoning

HW 28 Reactivity $\equiv \rho = \frac{k-1}{k}$, let us use k_∞ (Infinite reactor).

$$f_1 = \frac{\sum_a^{fuel}}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator}} \quad (\text{critical})$$


$$f_2 = \frac{\sum_a^{fuel}}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator} + \sum_a^{poison}}$$


Show that $\Delta\rho = \rho_2 - \rho_1 = -\frac{\sum_a^{poison}}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator}}$



Negative reactivity due to poison buildup. It is proportional to the amount of poison.

Poisoning

$$\frac{\partial I(\vec{r}, t)}{\partial t} = \gamma_I \sum_f \phi(\vec{r}, t) - \lambda_I I(\vec{r}, t) - \sigma_a^I I(\vec{r}, t) \phi(\vec{r}, t)$$


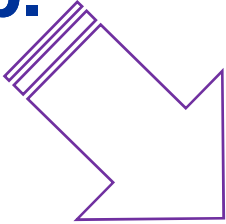
small

$$\frac{\partial Xe(\vec{r}, t)}{\partial t} = \gamma_{Xe} \sum_f \phi(\vec{r}, t) + \lambda_I I(\vec{r}, t) - \lambda_{Xe} Xe(\vec{r}, t) - \sigma_a^{Xe} Xe(\vec{r}, t) \phi(\vec{r}, t)$$

Initial conditions?


- **Clean Core Startup.**
- **Shutdown** (later).

Assume no spacial dependence.



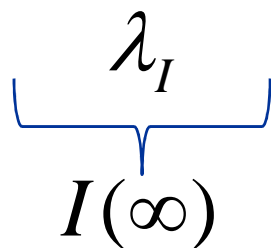
$I(0) = Xe(0) = 0$ Fresh Fuel.

and let us assume $\phi(t) = \phi(0) = \text{constant}$.



Poisoning

HW 29 Show that:
$$I(t) = \frac{\gamma_I \sum_f \phi_0}{\lambda_I} (1 - e^{-\lambda_I t})$$

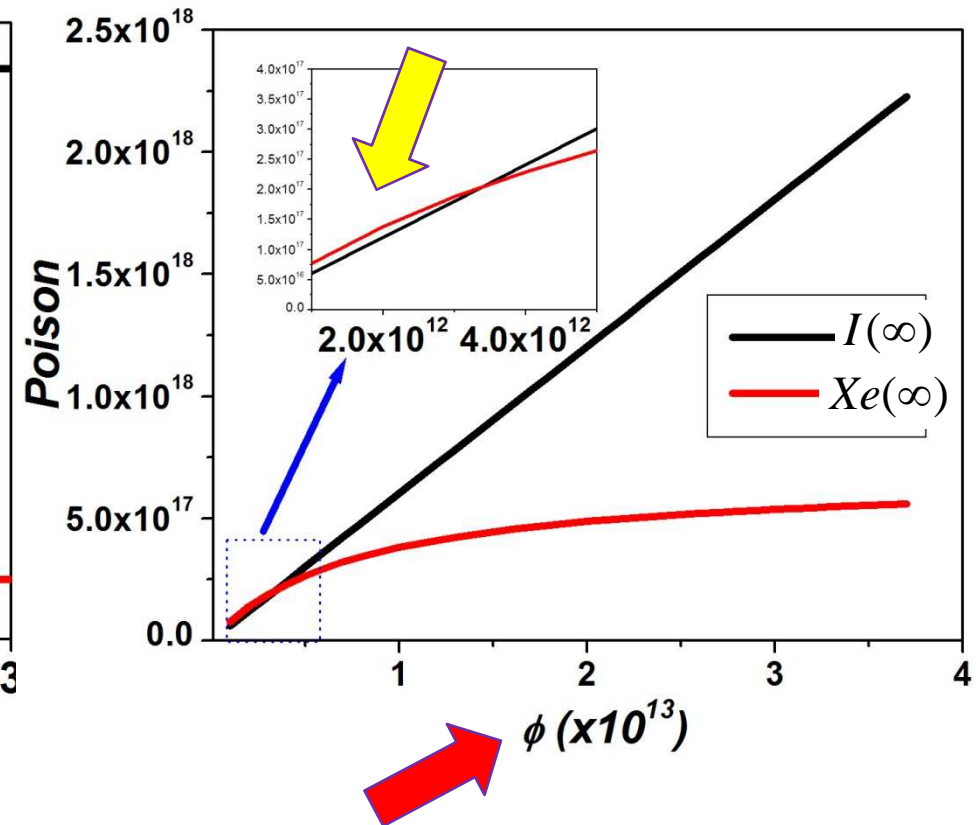
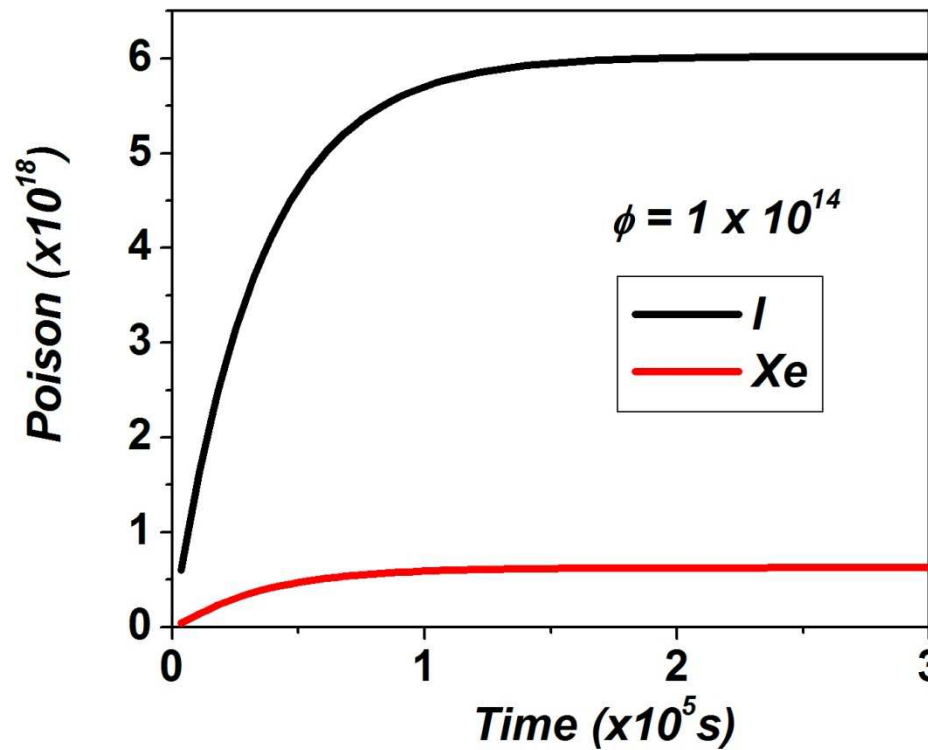


 $I(\infty)$

and
$$Xe(t) = \frac{\overbrace{(\gamma_I + \gamma_{Xe}) \sum_f \phi_0}^{Xe(\infty)}}{\lambda_{Xe} + \sigma_a^{Xe} \phi_0} (1 - e^{-(\lambda_{Xe} + \sigma_a^{Xe} \phi_0)t})$$

$$+ \frac{\gamma_I \sum_f \phi_0}{\lambda_{Xe} - \lambda_I + \sigma_a^{Xe} \phi_0} (e^{-(\lambda_{Xe} + \sigma_a^{Xe} \phi_0)t} - e^{-\lambda_I t})$$

Poisoning



- Now, we know $Xe(t)$

$$\Delta\rho = -\frac{\sum_a^{poison}(t)}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator}} = -\frac{\sigma_a^{Xe} Xe(t)}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator}}$$

Poisoning

- **Shutdown.** After the reactor has been operating for a “long” time.

$$\frac{\partial I(\vec{r}, t)}{\partial t} = \cancel{\gamma_I \sum_f \phi(\vec{r}, t)} - \lambda_I I(\vec{r}, t) - \cancel{\sigma_a^I I(\vec{r}, t) \phi(\vec{r}, t)}$$

$$\frac{\partial Xe(\vec{r}, t)}{\partial t} = \cancel{\gamma_{Xe} \sum_f \phi(\vec{r}, t)} + \lambda_I I(\vec{r}, t) - \lambda_{Xe} Xe(\vec{r}, t) - \cancel{\sigma_a^{Xe} Xe(\vec{r}, t) \phi(\vec{r}, t)}$$

$$I(0) = I(\infty)$$

$$Xe(0) = Xe(\infty)$$

$$\phi(t) = \phi(0) = 0.$$

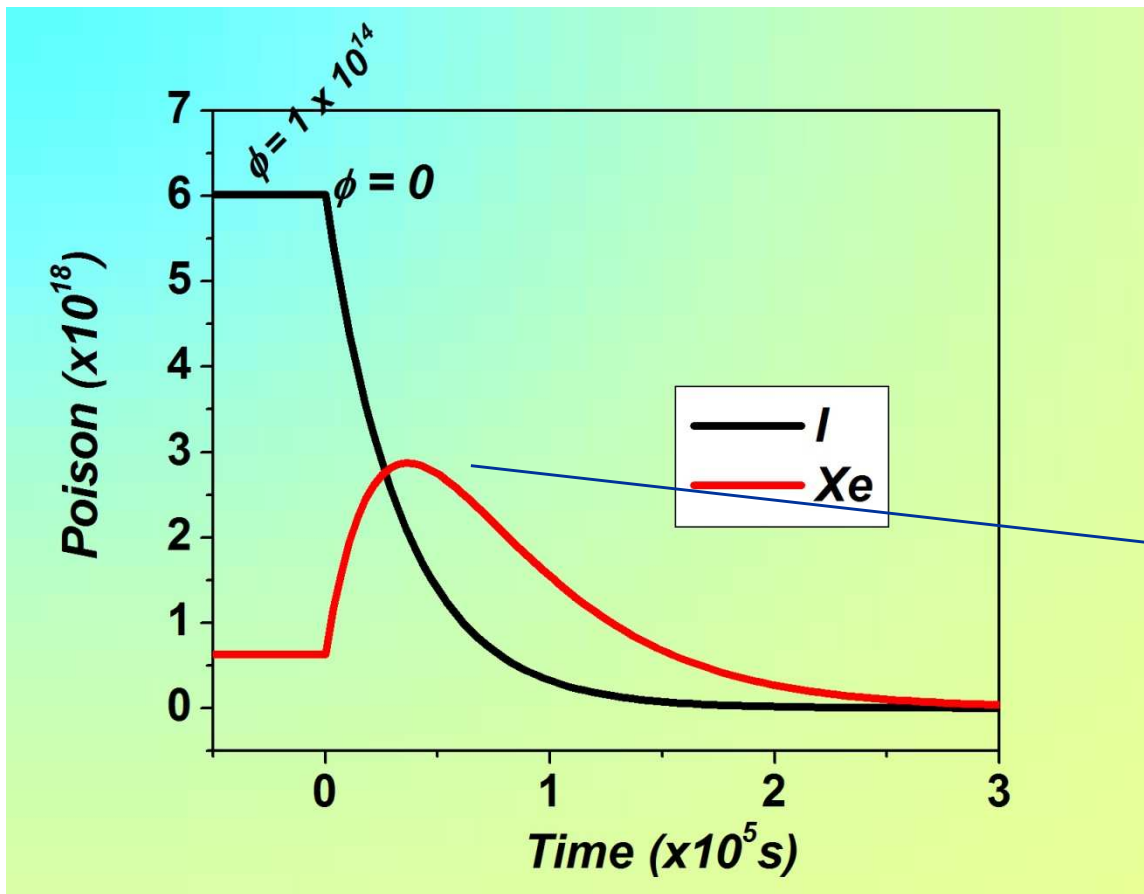
$$\frac{\partial I(\vec{r}, t)}{\partial t} = -\lambda_I I(\vec{r}, t)$$

$$\frac{\partial Xe(\vec{r}, t)}{\partial t} = \lambda_I I(\vec{r}, t) - \lambda_{Xe} Xe(\vec{r}, t)$$

Poisoning

HW 30 Show that $I(t) = I(\infty)e^{-\lambda_I t}$

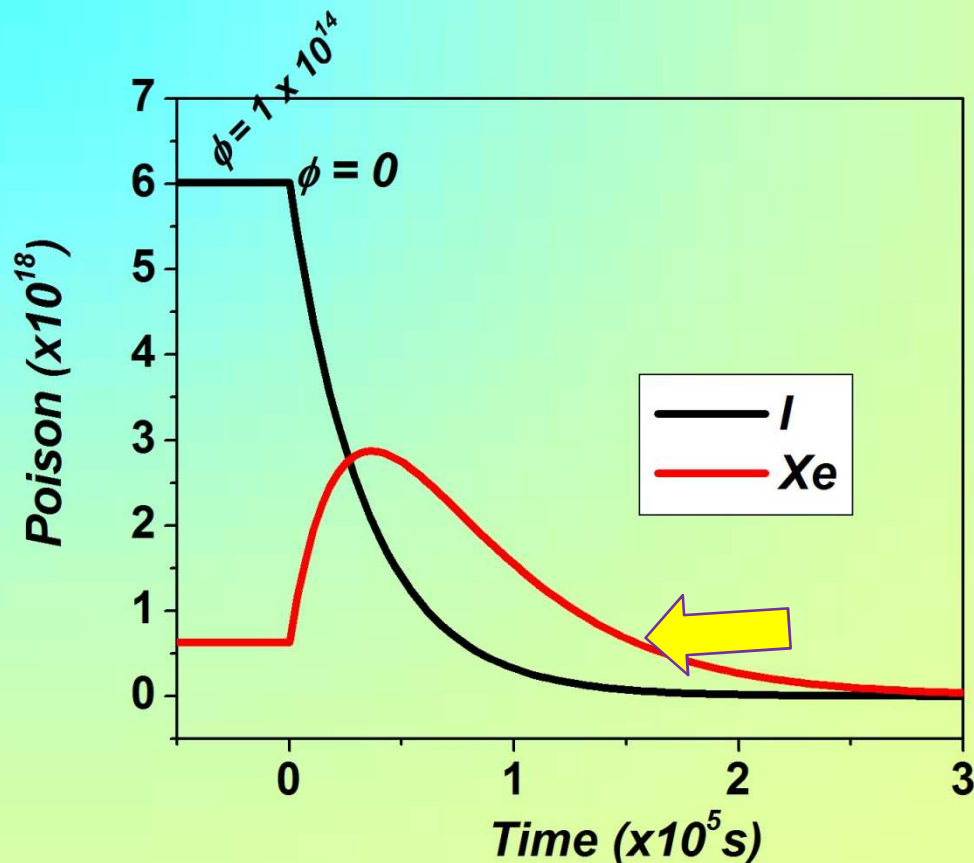
$$Xe(t) = Xe(\infty)e^{-\lambda_{Xe}t} + \underbrace{\frac{\lambda_I I(\infty)}{\lambda_I - \lambda_{Xe}}}_{> 0?} (e^{-\lambda_{Xe}t} - e^{-\lambda_I t})$$



Height of the peak depends on $I(\infty)$ and $Xe(\infty)$, i.e. depends on ϕ .

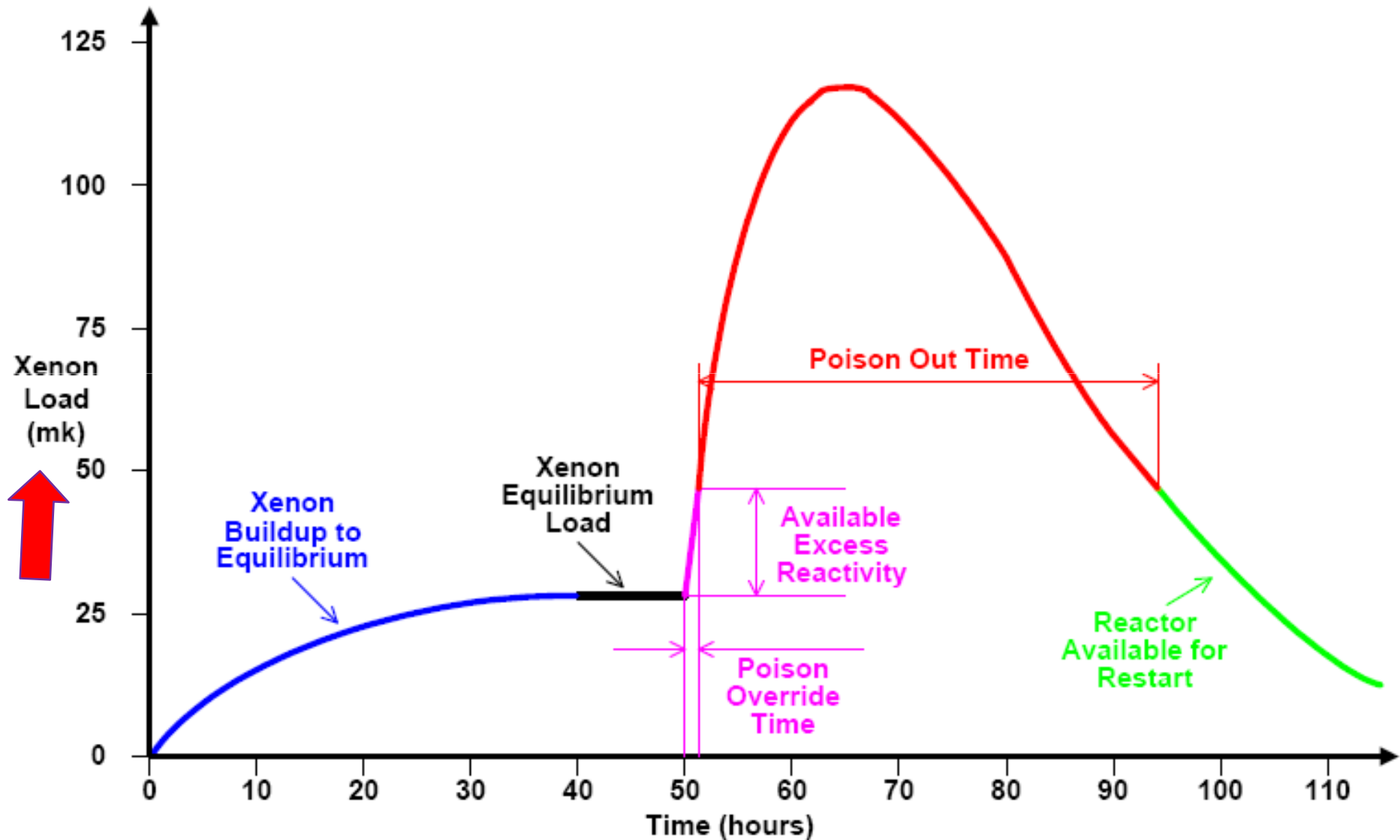
Poisoning

Shutdown ► Xe ↑ ► negative $\Delta\rho$ ► try to add positive reactivity ► move control rods out ► need to have enough reserve ► **costly to do that.**



If, the available excess reactivity can compensate for less than 30 minutes of poison buildup, can't startup again after ~30 minutes of shutdown, because you can't achieve criticality. Need to wait some 40 hours (in this case) for Xe to decay down.

Poisoning



Poisoning

Strategies

- If you plan to shut down for “short maintenance”, think about stepback.
- Examine different scenarios using this [code](http://www.nuceng.ca/) from <http://www.nuceng.ca/>
- Prepare your own report, code, calculations, graphs, comments, conclusions etc.....
- Be creative and use whatever experience you gained during your study in this program.

**20% (8 marks) of
the final exam.**

Poisoning

Xe Oscillations

- $\phi(\mathbf{r}, t)$ (spacial dependence) \blacktriangleright flux \uparrow locally \blacktriangleright Xe burnup \uparrow \blacktriangleright ρ (reactivity) \uparrow \blacktriangleright flux further \uparrow \blacktriangleright control rods globally in \blacktriangleright flux \downarrow elsewhere \blacktriangleright Xe burnup \downarrow \blacktriangleright ρ \downarrow Xe oscillation but limited by opposite effect due to increase (decrease) of I in the high (low) flux region.
- In large reactors (compared to neutron diffusion length) local flux, power and temperature could reach unacceptable values for certain materials \blacktriangleright safety issues.
- Think of one sensor and one control rod \blacktriangleright feel average flux \blacktriangleright apparently OK \blacktriangleright more sensors and control rods to locate and deal with local changes.

Poisoning

Permanent Poisons

- ^{149}Sm has sizeable but lower cross section than ^{135}Xe .
- It does not decay.

$$\frac{\partial \text{Sm}(\vec{r}, t)}{\partial t} \approx \gamma_{\text{Sm}} \sum_f (\vec{r}, t) \phi(\vec{r}, t) \dots \text{?????} \dots$$

- Accumulates with time.
- Consequences?????????