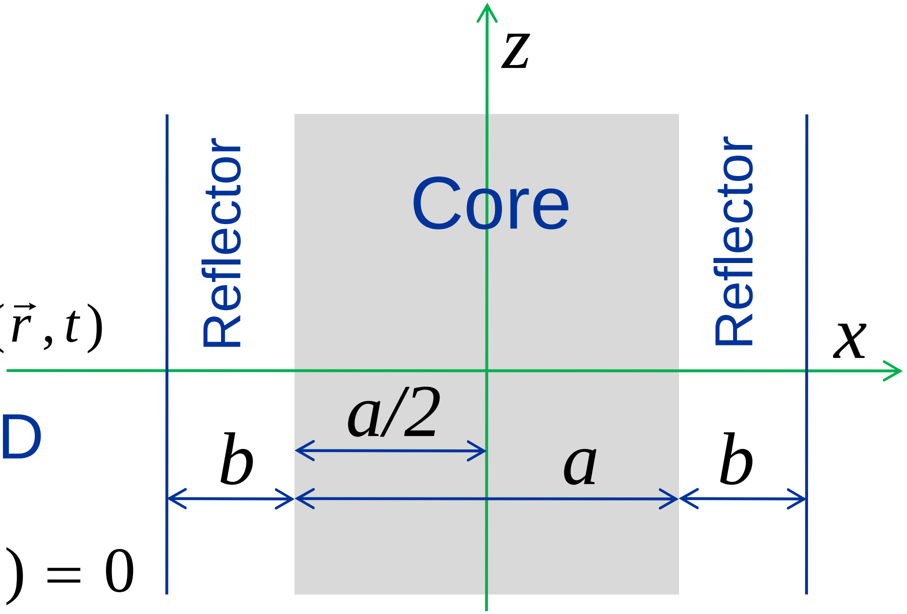
Reflected Slab: One-Group

Reflected Slab Reactor

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = \nu\Sigma_f(\vec{r})\phi(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla}\bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$



For steady-state, homogeneous, 1-D

$$D^C\frac{d^2\phi^C(x)}{dx^2} + (\nu^C\Sigma_f^C - \Sigma_a^C)\phi^C(x) = 0$$

C ≡ Core

$$D^R\frac{d^2\phi^R(x)}{dx^2} - \Sigma_a^R\phi^R(x) = 0$$

R ≡ Reflector

Recall:

$$\nabla^2\phi(\vec{r}) - \frac{1}{L^2}\phi(\vec{r}) = 0$$

BCs $\quad \phi^R(\frac{a}{2}+b)=0, \quad \phi^R(\frac{a}{2})=\phi^C(\frac{a}{2}), \quad J^R(\frac{a}{2})=J^C(\frac{a}{2})$

Reflected Slab: One-Group

$$\phi^C(x) = A^C \cos(B_m^C x) \qquad (B_m^C)^2 = \frac{\nu \Sigma_f^C - \Sigma_a^C}{D^C}$$

Verify.

$$\phi^R = A^R \sinh\left[\frac{\frac{a}{2}+b-x}{L^R}\right] \qquad (L^R)^2 = \frac{D^R}{\Sigma_a^R}$$

BC

$$A^C \cos\left(\frac{B_m^C a}{2}\right) = A^R \sinh\left[\frac{b}{L^R}\right]$$

$$D^C B_m^C A^C \sin\left(\frac{B_m^C a}{2}\right) = \frac{D^R}{L^R} A^R \cosh\left[\frac{b}{L^R}\right]$$

$$D^C B_m^C \tan\left(\frac{B_m^C a}{2}\right) = \frac{D^R}{L^R} \coth\left[\frac{b}{L^R}\right]$$

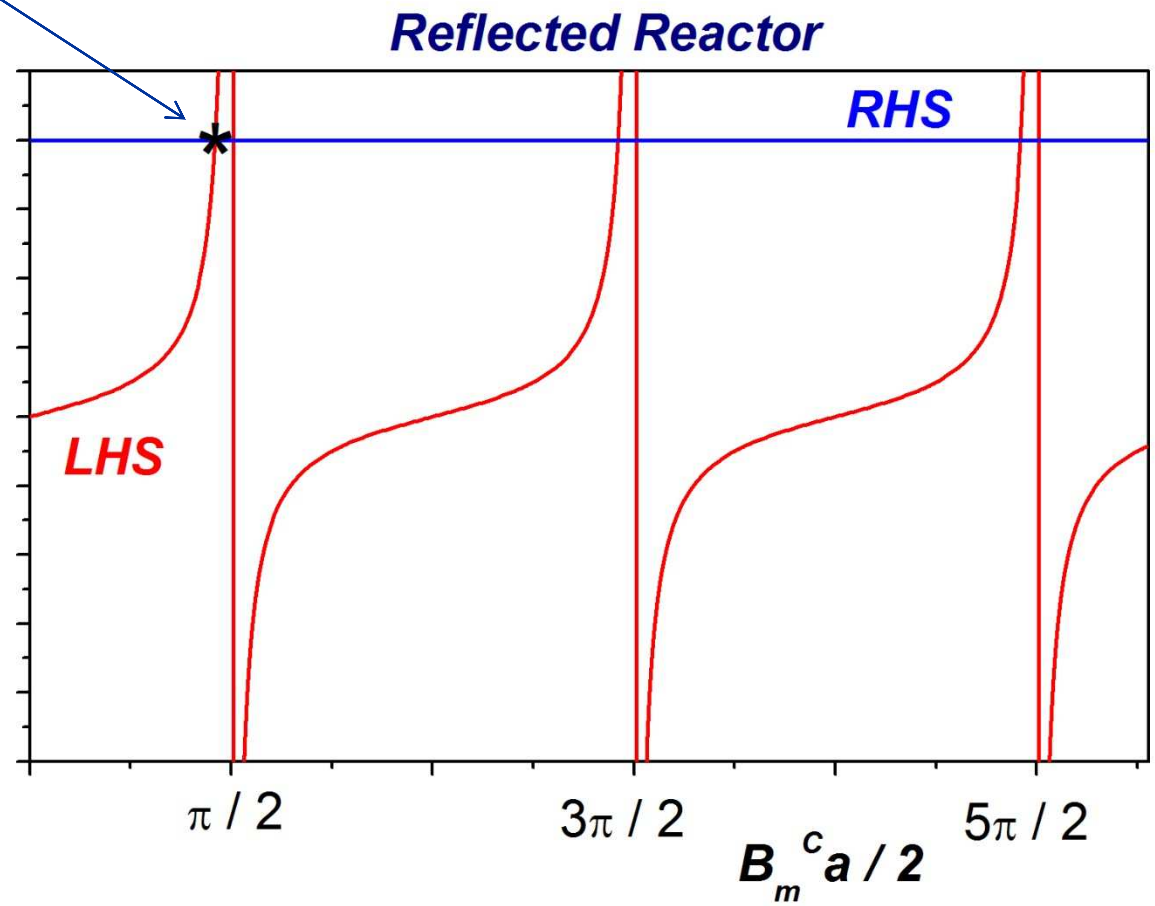
Criticality condition.

Reflected Slab: One-Group

Criticality condition.

$$D^C B_m^C \tan\left(\frac{B_m^C a}{2}\right) = \frac{D^R}{L^R} \coth\left[\frac{b}{L^R}\right]$$

- For bare slab CC was $\pi / 2$.
- Smaller core for reflected reactor (even with $a_0 > a$).
- Save fuel.



Criticality "Calculation"

- Can we solve "real" reactor problems analytically?
- The previous discussion provides understanding of the concepts but also indicates the need for **computational techniques.**

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = \nu \Sigma_f(\vec{r})\phi(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla} \bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

- Assume: $\phi(\vec{r},t) = e^{\lambda t}\psi(\vec{r})$

▼

$$\frac{\lambda}{v}\psi(\vec{r}) = \nu \Sigma_f(\vec{r})\psi(\vec{r}) - \Sigma_a(\vec{r})\psi(\vec{r}) + \vec{\nabla} \bullet D(\vec{r})\vec{\nabla}\psi(\vec{r})$$

- Adjust parameters so that λ = 0 (Steady-state).
- What parameters and how to adjust them?

Criticality "Calculation"

$$\frac{\lambda}{v}\psi(\vec{r}) = \nu\Sigma_f(\vec{r})\psi(\vec{r}) - \Sigma_a(\vec{r})\psi(\vec{r}) + \vec{\nabla}\bullet D(\vec{r})\vec{\nabla}\psi(\vec{r})$$

- Fixed design and geometry ▶ one free variable is *k*

$$-\vec{\nabla}\bullet D(\vec{r})\vec{\nabla}\psi(\vec{r}) + \Sigma_a(\vec{r})\psi(\vec{r}) = \frac{\nu}{k_{fudge}}\Sigma_f(\vec{r})\psi(\vec{r})$$

$$M\psi = \frac{1}{K_{fudge}}F\psi \qquad M, F \text{ are operators}$$

- **As we did earlier (be guided by HW 20):**

$$k_{fudge} = \frac{\nu\Sigma_f\psi}{-\vec{\nabla}\bullet D\vec{\nabla}\psi + \Sigma_a\psi} = \frac{\nu\Sigma_f/\Sigma_a}{1 + B^2L^2}$$

Criticality "Calculation"

$$k_{fudge} = \frac{\nu \Sigma_f / \Sigma_a}{1 + B^2 L^2} \qquad M\psi = \frac{1}{K_{fudge}} F\psi$$

- Build an algorithm.
- "Guess" (reasonably) initial k_{fudge} and ψ (or ϕ) for the zeroth iteration.
- Calculate the initial source term.
- Iterate:

Guess ϕ^0 and k^0.

$$M\phi^1 = \frac{1}{k^0} F\phi^0 = \frac{S^0}{k^0} \Rightarrow \text{get } \phi^1$$

$$S^1 = F\phi^1$$

$$k^1 = \frac{k^0 S^1}{S^0}$$

and so on ..... until flux converges.

Criticality "Calculation"

- Or:

$$k = \frac{\text{fission sources}}{\text{sinks}} = \frac{F\phi}{M\phi}$$

$$k^{i+1} = \frac{\int_{volume} F\phi^{i+1} dV}{\int_{volume} M\phi^{i+1} dV} = \frac{\int_{volume} S^{i+1} dV}{\frac{1}{k^i}\int_{volume} S^i dV}$$

- If for example $k > 1$, take action to reduce source or increase absorption.
- **How?**

How to Adjust Criticality

Reactor Kinetics

Reactor kinetics refers to the manipulation of parameters that affect k and to the subsequent direct response of the reactor system. Examples are:

- Absorber rods or shim movements to compensate for fuel burnup.
- Safety scram rods to rapidly shutdown the chain reaction.
- Control rods to provide real-time control to keep k = 1 or to maneuver up and down in power.
- …..

Negative or positive **reactivity.**

Reactor Dynamics

Reactor dynamics refers to the more indirect feedback mechanisms due to power level effects and other overall system effects such as:

- Temperature feedback.
- **Void** feedback.
- Pump speed control (affects water density and temperature).
- …

How to Adjust Criticality

Before all:

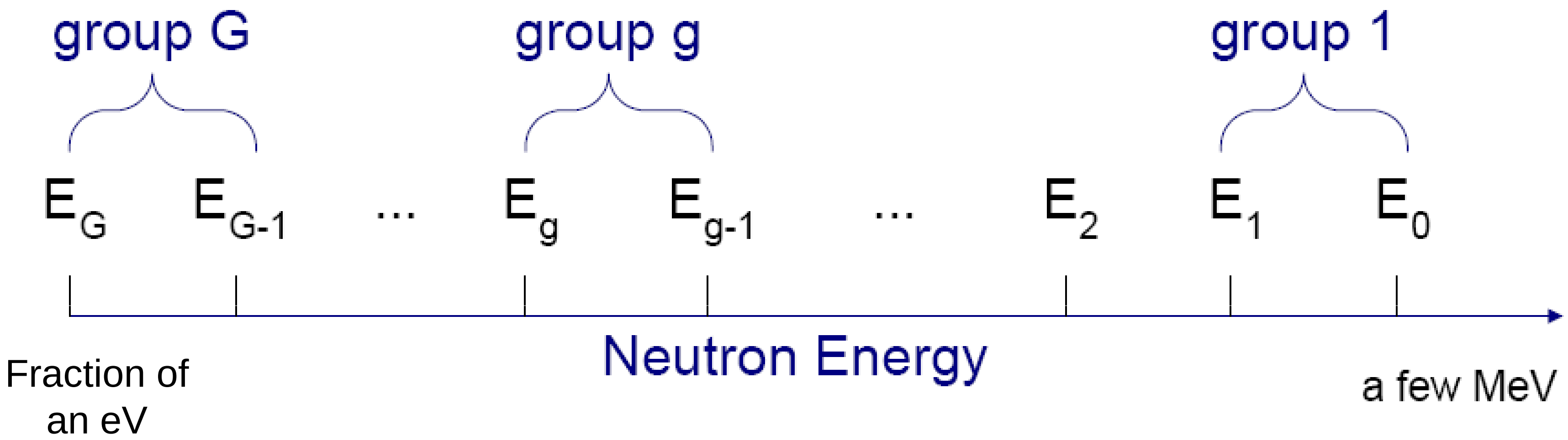
Core Design

The transient response of the reactor to the above direct and indirect changes in basic parameters is highly dependent on the design details of the reactor. Sample issues are:

- Where should the control rods be placed for maximum effectiveness?
- Will the power go up or down if a void is introduced into the reactor?
- Will the power go up or down if core temperature goes up?
- How often should the reactor be refueled?
- and so on...

Multi-group Model

- Wide neutron spectrum.
- One-group, two-group? Should be generalized.



$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$

$$-\Sigma_{ag}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

Flux-averaged quantities.

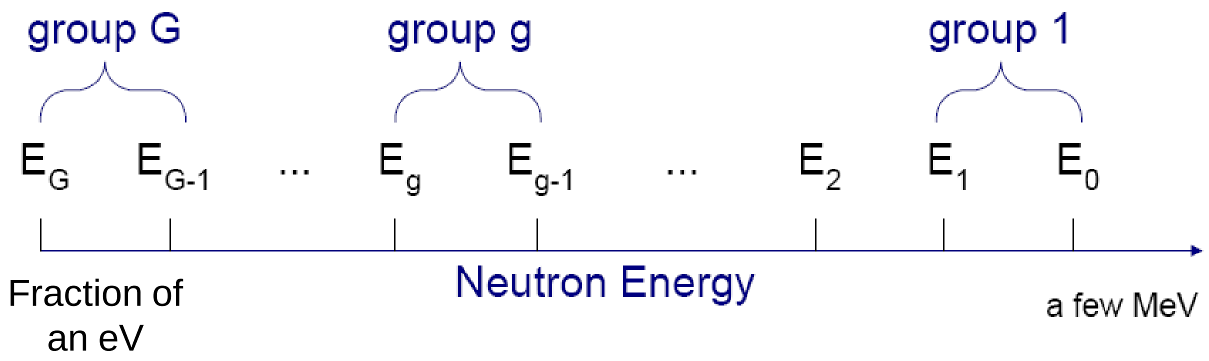
$$\phi_g(\vec{r},t) \equiv \int_{E_g}^{E_{g-1}} \phi(\vec{r},E,t)\,dE$$

Identify the terms, NOW.

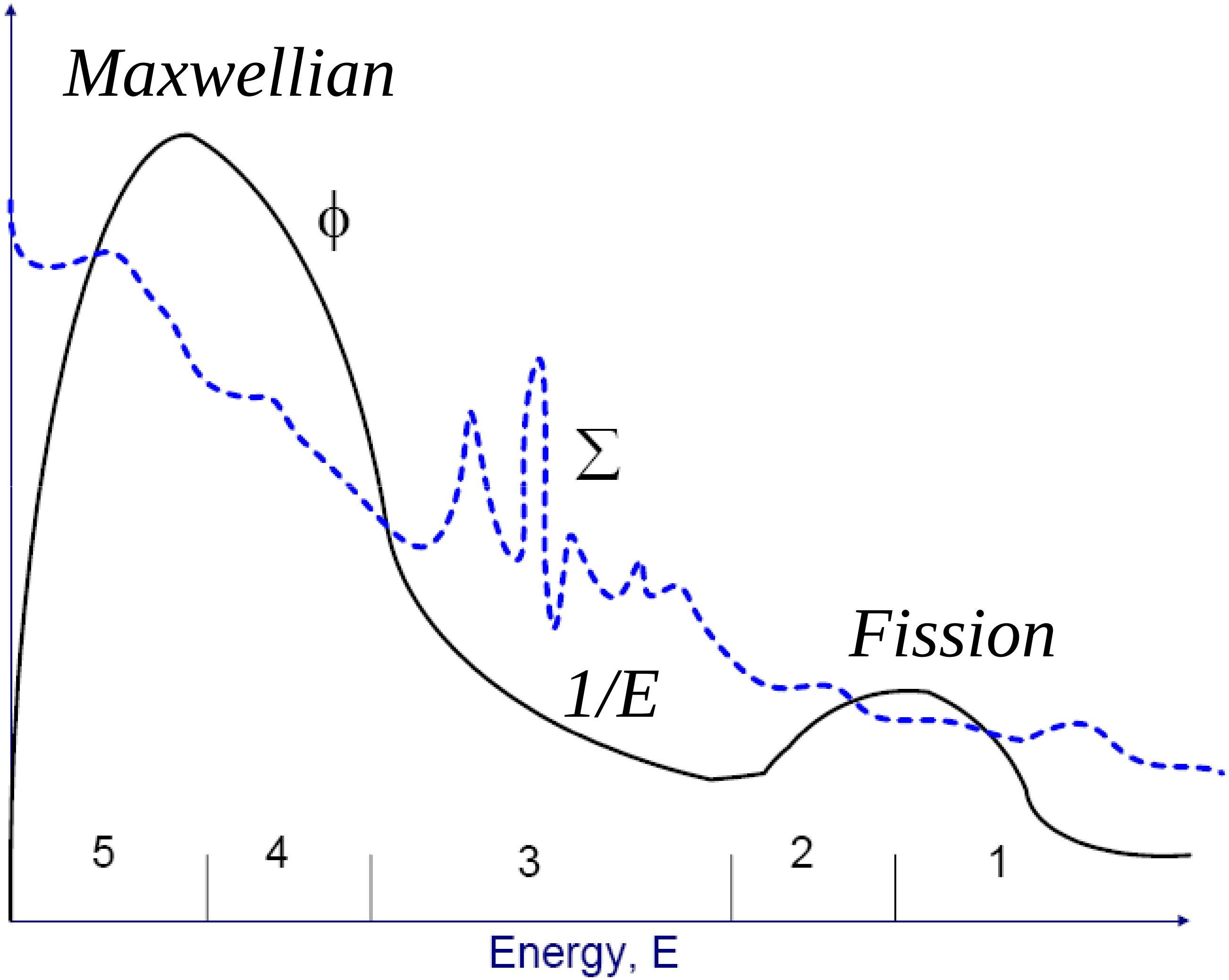
Multi-group Model

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \overbrace{\chi_g}^{\text{Fraction}} \overbrace{\sum_{g^\backslash=1}^{G} \nu_{g^\backslash} \Sigma_{fg^\backslash}(\vec{r})\phi_{g^\backslash}(\vec{r},t)}^{\text{Total fission}} + \overbrace{\sum_{g^\backslash=1}^{G} \Sigma_{sg^\backslash g}(\vec{r})\phi_{g^\backslash}(\vec{r},t)}^{\text{Scattering in}} + \overbrace{S_g^{ext}}^{\text{Other sources}}$$

$$\underbrace{-\Sigma_{ag}(\vec{r})\phi_g(\vec{r},t)}_{\text{Absorption}} \underbrace{-\Sigma_{sg}(\vec{r})\phi_g(\vec{r},t)}_{\text{Scattering out}} + \underbrace{\vec{\nabla}\bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)}_{\text{Leakage}}$$



Multi-group Model



5-group example.

Multi-group Model

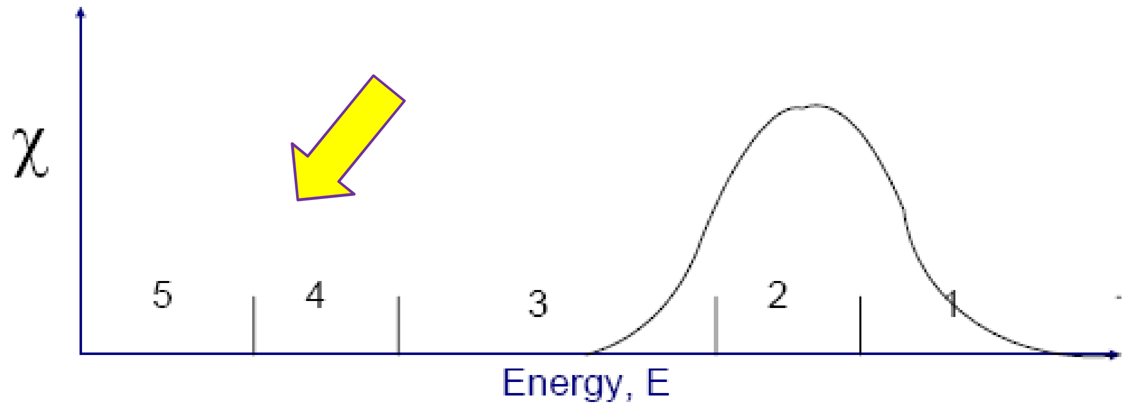
Total fission

$$\chi_g \sum_{g'=1}^{5} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) =$$

$$\chi_g \left[\nu_5 \Sigma_{f5} \phi_5 + \nu_4 \Sigma_{f4} \phi_4 + \nu_3 \Sigma_{f3} \phi_3 + \nu_2 \Sigma_{f2} \phi_2 + \nu_1 \Sigma_{f1} \phi_1\right]$$

Thermal fission (~ 97%)

Fast fission (~ 3%)



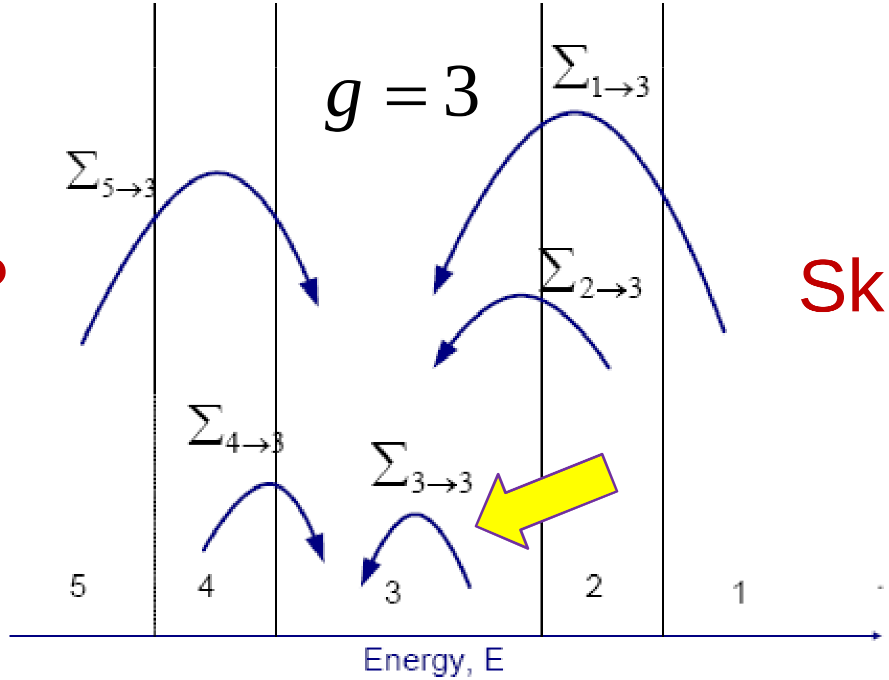
Multi-group Model

Scattering in

$$\sum_{g'=1}^{5} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) =$$

$$\Sigma_{s1g}\,\phi_1 + \Sigma_{s2g}\,\phi_2 + \Sigma_{s3g}\,\phi_3 + \Sigma_{s4g}\,\phi_4 + \Sigma_{s5g}\,\phi_5$$

Upscattering!!???



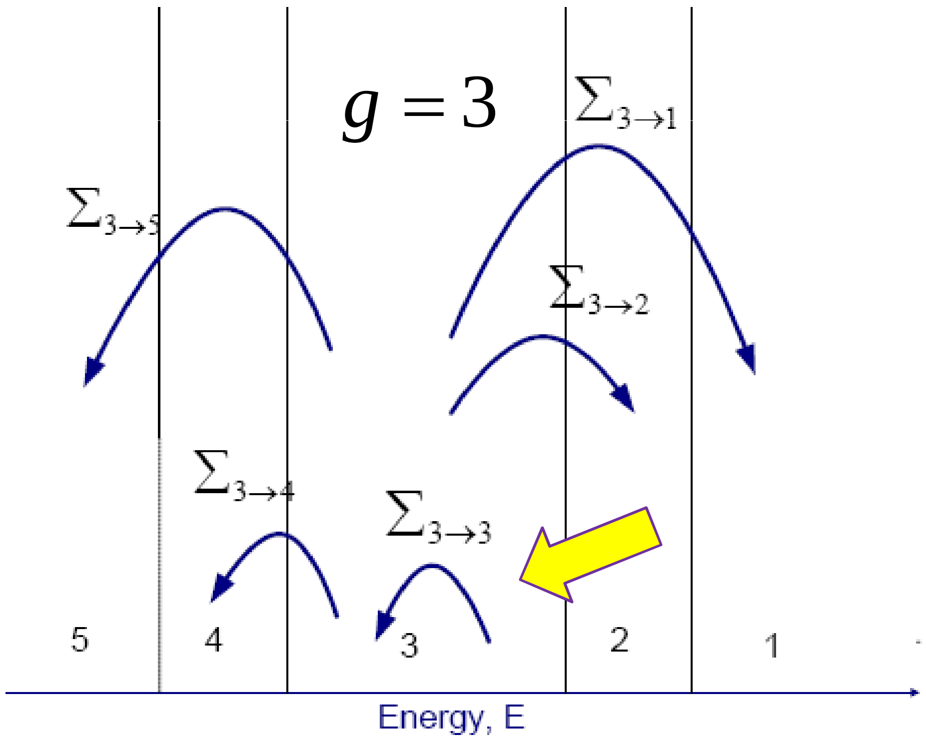
Skipping!!???

Multi-group Model

Scattering out

$$\Sigma_{sg}(\vec{r})\phi_g(\vec{r},t) =$$

$$\Sigma_{sg1}\phi_g + \Sigma_{sg2}\phi_g + \Sigma_{sg3}\phi_g + \Sigma_{sg4}\phi_g + \Sigma_{sg5}\phi_g$$



Multi-group Model

Group 3

$$\frac{1}{v_3}\frac{\partial}{\partial t}\phi_3(\vec{r},t) = \chi_3 \sum_{g^\backslash=1}^{5} \nu_{g^\backslash} \Sigma_{fg^\backslash}(\vec{r})\phi_{g^\backslash}(\vec{r},t) + S_3^{ext}$$

$$- \Sigma_{a3}(\vec{r})\phi_3(\vec{r},t) + \vec{\nabla} \bullet D_3(\vec{r})\vec{\nabla}\phi_3(\vec{r},t)$$

$$- \left[\Sigma_{s31}\phi_3 + \Sigma_{s32}\phi_3 + \cancel{\Sigma_{s33}\phi_3} + \Sigma_{s34}\phi_3 + \Sigma_{s35}\phi_3\right]$$

$$+ \left[\Sigma_{s13}\phi_1 + \Sigma_{s23}\phi_2 + \cancel{\Sigma_{s33}\phi_3} + \Sigma_{s43}\phi_4 + \Sigma_{s53}\phi_5\right]$$

Removal cross section

$$\Sigma_{r3} \equiv \Sigma_{a3} + \Sigma_{s3} - \Sigma_{s33}$$

$$= \Sigma_{a3} + \Sigma_{s31} + \Sigma_{s32} + \Sigma_{s34} + \Sigma_{s35}$$

$$= \Sigma_{a3} + \sum_{\substack{g^\backslash=1 \\ g^\backslash\neq 3}}^{5} \Sigma_{s3g^\backslash}$$

Multi-group Model

Fraction

Total fission

Scattering in

Other sources

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{g'=1}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$

$$- \Sigma_{rg}(\vec{r})\phi_g(\vec{r},t) - \Sigma_{sgg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$

Removal

In-group Scattering

Leakage

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\vec{r},t) = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r},t) + \sum_{\substack{g'=1 \\ g'\neq g}}^{G} \Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r},t) + S_g^{ext}$$

Net Scattering in

$$- \Sigma_{rg}(\vec{r})\phi_g(\vec{r},t) + \vec{\nabla} \bullet D_g(\vec{r})\vec{\nabla}\phi_g(\vec{r},t)$$