Back to Multiplication Factor

Recall:

Things to be used later…!

$$k_\infty = fp\varepsilon\eta, \quad \frac{k_{eff}}{k_\infty} = P_{non-leak} \quad ▶ \quad k_{eff} = f\rho\varepsilon\eta P_{non-leak}$$

- Fast from thermal, $\eta = \frac{1}{\Sigma_a} \sum_i \nu(i) \Sigma_f(i)$
- Fast from fast, ε.
- Thermal from fast, p.
- Thermal available for fission $f = \frac{\sum_a^{fuel}}{\sum_a^{fuel} + \sum_a^{clad} + \sum_a^{moderator} + \sum_a^{poison}}$

Recall:

Thinking QUIZ

- For each thermal neutron absorbed, how many fast neutrons are produced?

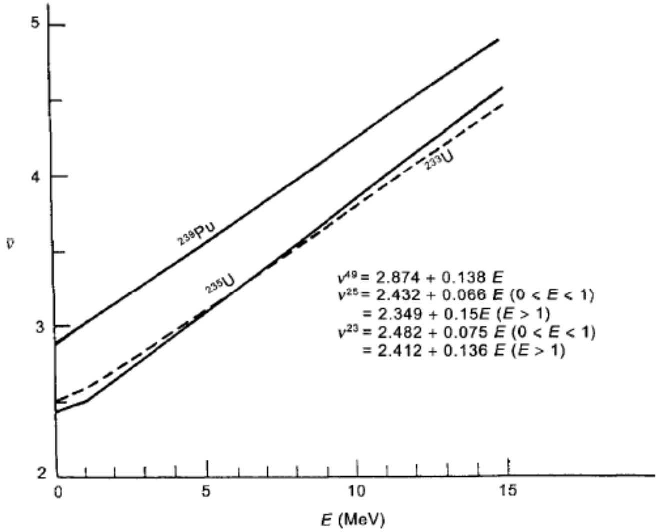
Two-Group Neutron Diffusion

- **Introductory to multi-group (Hence crude).**
- All neutrons are either in a fast or in a thermal energy group.
- Boundary between two groups is set to ~1 eV.
- **Thermal neutrons** diffuse in a medium and cause fission, are captured, or leak out from the system.
- Source for thermal neutrons is provided by the slowing down of fast neutrons (born in fission).
- **Fast neutrons** are lost by slowing down due to elastic scattering in the medium, or leak out from the system, or due to fission or capture.
- Source for fast neutrons is thermal and fast neutron fission.

Two-Group Neutron Diffusion

$$\phi_1(\vec{r}) = \int_{1eV}^{10MeV} \phi(E,\vec{r})dE \qquad \textit{Fast}$$

$$\phi_2(\vec{r}) = \int_{0}^{1eV} \phi(E,\vec{r})dE \qquad \textit{Thermal}$$



$$k_{eff} = \frac{\nu_1 \Sigma_{f1} \phi_1 + \nu_2 \Sigma_{f2} \phi_2}{-D_1 \nabla^2 \phi_1 - D_2 \nabla^2 \phi_2 + \Sigma_{a1} \phi_1 + \Sigma_{a2} \phi_2}$$

Two-Group Neutron Diffusion

Fast

$$0 = S_1(\vec{r}) - \Sigma_{a1}\,\phi_1(\vec{r}) + D_1\nabla^2\phi_1(\vec{r})$$

Depends on thermal and fast fluxes.

Removal cross section = fission + capture + scattering to group 2

Fast diffusion coefficient

$$0 = \nu\,\Sigma_{f1}\,\phi_1(\vec{r}) + \nu\,\Sigma_{f2}\,\phi_2(\vec{r}) - \Sigma_{a1}\,\phi_1(\vec{r}) + D_1\nabla^2\phi_1(\vec{r})$$

or

$$0 = \frac{k_\infty}{\rho}\Sigma_{a2}\,\phi_2(\vec{r}) - \Sigma_{a1}\,\phi_1(\vec{r}) + D_1\nabla^2\phi_1(\vec{r})$$

Two-Group Neutron Diffusion

Thermal

$$0 = S_2(\vec{r}) - \Sigma_{a2}\,\phi_2(\vec{r}) + D_2\nabla^2\phi_2(\vec{r})$$

Depends on fast flux.

Thermal absorption cross section = fission + capture.

Thermal diffusion coefficient

$$0 = \Sigma_{s1\to 2}\,\phi_1(\vec{r}) - \Sigma_{a2}\,\phi_2(\vec{r}) + D_2\nabla^2\phi_2(\vec{r})$$

or

$$0 = \rho\,\Sigma_{a1}\,\phi_1(\vec{r}) - \Sigma_{a2}\,\phi_2(\vec{r}) + D_2\nabla^2\phi_2(\vec{r})$$

Two-Group Neutron Diffusion

$$0 = \frac{k_\infty}{\rho} \Sigma_{a2}\, \phi_2(\vec{r}) - \Sigma_{a1}\, \phi_1(\vec{r}) + D_1 \nabla^2 \phi_1(\vec{r})$$

$$0 = \rho \Sigma_{a1}\, \phi_1(\vec{r}) - \Sigma_{a2}\, \phi_2(\vec{r}) + D_2 \nabla^2 \phi_2(\vec{r})$$

- A coupled system of equations; both depend on both fluxes.
- Recall also, for a steady state system:

$$\nabla^2 \phi_1(\vec{r}) + B^2 \phi_1(\vec{r}) = 0$$

$$\nabla^2 \phi_2(\vec{r}) + B^2 \phi_2(\vec{r}) = 0$$

Geometrical

Two-Group Neutron Diffusion

Homogeneous system ▶ Determinant of coefficients matrix = 0

$$\begin{vmatrix} -\Sigma_{a1} - D_1 B^2 & \dfrac{k_\infty}{\rho}\Sigma_{a2} \\ \rho\Sigma_{a1} & -\Sigma_{a2} - D_2 B^2 \end{vmatrix} = 0$$

Review Cramer's rule!
Do we need it here?

$$(-\Sigma_{a1} - D_1 B^2)(-\Sigma_{a2} - D_2 B^2) - \frac{k_\infty}{\rho}\Sigma_{a2}\,\rho\Sigma_{a1} = 0$$

$$(\Sigma_{a1} + D_1 B^2)(\Sigma_{a2} + D_2 B^2) - k_\infty \Sigma_{a2}\Sigma_{a1} = 0$$

$$(\frac{1}{L_{Fast}^2} + B^2)(\frac{1}{L_{Thermal}^2} + B^2) - k_\infty \frac{1}{L_{Fast}^2}\frac{1}{L_{Thermal}^2} = 0$$

$$(1 + B^2 L_{Fast}^2)(1 + B^2 L_{Thermal}^2) - k_\infty = 0$$

Two-Group Neutron Diffusion

$$(1+B^2L^2_{Fast})(1+B^2L^2_{Thermal})-k_\infty=0$$

$$\frac{k_\infty}{(1+B^2L^2_{Fast})(1+B^2L^2_{Thermal})}=1$$

k_{eff} for a critical reactor

$$\frac{k_{eff}}{k_\infty}=P^{Fast}_{non-leak}P^{Thermal}_{non-leak}=\frac{1}{B^2L^2_{Thermal}+1}\frac{1}{B^2L^2_{Fast}+1}$$

For large reactors

$$\frac{k_\infty}{1+B^2\underbrace{(L^2_{Fast}+L^2_{Thermal})}}=1\Rightarrow B^2=\frac{k_\infty-1}{M^2}$$

Migration Length

Two-Group Neutron Diffusion

$$M^2 = L^2_{Thermal} + L^2_{Fast}$$

If any ↑ leakage ↑.

$$L^2_{Thermal} = \frac{D}{\Sigma_a} = \frac{\lambda_{tr}}{3\Sigma_a} = \frac{1}{3\Sigma_a \Sigma_{tr}}$$

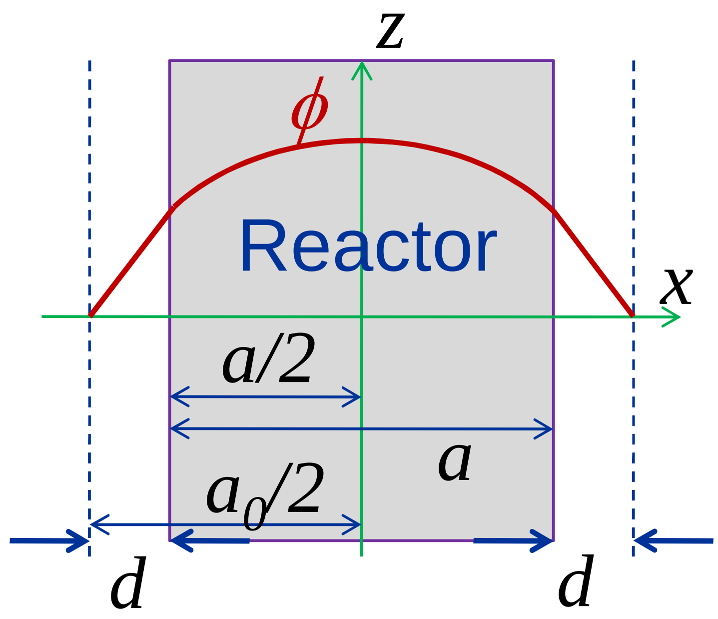
$$\text{Fermi age} \equiv L^2_{Fast} = \frac{n}{3\Sigma_s \Sigma_{tr}}$$

- Slowing down density.
- Fermi model.

Do it.

Reactor Model: One-Group

- Before considering multi-group.
- So far we did 1-D.
- Back to **one-group** but extend to **3-D**.



1-D

HW 21

For the homogeneous infinite slab reactor, extend the criticality condition that you found in HW 21.

$$B_g^2 = \left(\frac{\pi}{a_0}\right)^2 = B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{\nu \Sigma_f - \Sigma_a}{D}$$

Reactor Model: One-Group

- In 3-D

$$\frac{d^2\phi(x)}{dx^2} + B^2\phi(x) = 0 \quad \blacktriangleright \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} + B^2\phi = 0$$

$$B^2 \leftarrow \frac{\nu\Sigma_f - \Sigma_a}{D}$$

$$\phi = \phi_0 \cos Bx \quad \blacktriangleright \quad \phi = \phi_0 \cos B_x x \cos B_y y \cos B_z z$$

$$B_g^2 = \left(\frac{\pi}{a_0}\right)^2 = B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{\nu\Sigma_f - \Sigma_a}{D} \quad \blacktriangleright$$

$$B_g^2 = B_x^2 + B_y^2 + B_z^2 = \underbrace{\left(\frac{\pi}{a_0}\right)^2 + \left(\frac{\pi}{b_0}\right)^2 + \left(\frac{\pi}{c_0}\right)^2}_{} = B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{\nu\Sigma_f - \Sigma_a}{D}$$

Critical dimensions (size), for the given material properties, predicted by the model.

Reactor Model: One-Group

- Transient case.

t !

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla}\bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

t ! $\Sigma_a^{fuel} = \Sigma_f^{fuel} + \Sigma_\gamma^{fuel}$

Moderator, structure, coolant, fuel, …

- Delayed neutrons!!
- Reflectors!!
- For homogeneous 1-D:

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(x,t) = S(x,t) - \Sigma_a\phi(x,t) + D\frac{\partial^2}{\partial x^2}\phi(x,t)$$

$$\nu\Sigma_f\phi(x,t)$$

Reactor Model: One-Group

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(x,t) = \nu\Sigma_f\,\phi(x,t) - \Sigma_a\,\phi(x,t) + D\frac{\partial^2}{\partial x^2}\phi(x,t)$$

HW 25

Separation of variables: $\phi(x,t) = \psi(x)T(t)$

$$\frac{1}{v}\psi\frac{\partial T}{\partial t} = \nu\Sigma_f\,\psi T - \Sigma_a\,\psi T + DT\frac{\partial^2\psi}{\partial x^2}$$

$$\frac{1}{T}\frac{\partial T}{\partial t} = \frac{v}{\psi}\left[D\frac{\partial^2\psi}{\partial x^2} + (\nu\Sigma_f - \Sigma_a)\psi\right] \equiv -\lambda = \text{constant}$$

= 0 for steady state.

Show that $T(t) = T(0)e^{-\lambda t}, \quad \lambda = v(\Sigma_a + DB^2 - \nu\Sigma_f)$

Reactor Model: One-Group

HW 25 (continued)

$$\phi(\pm\frac{a_0}{2}) = 0$$ try $$\psi_n(x) = \cos B_n x \qquad B_n^2 = \left(\frac{n\pi}{a_0}\right)^2$$

eigenvalues $$\lambda_n = v(\Sigma_a + DB_n^2 - \nu\Sigma_f)$$

Solution $$\phi(x,t) = \sum_{n \quad odd} A_n e^{-\lambda_n t}\cos\left(\frac{n\pi x}{a_0}\right)$$

? ?

Initial condition $$\phi(x,0) = \sum_{n \quad odd} A_n \cos\left(\frac{n\pi x}{a_0}\right)$$

Show that $$A_n = \frac{2}{a_0}\int_{-\frac{a_0}{2}}^{+\frac{a_0}{2}} \phi(x,0)\cos\left(\frac{n\pi x}{a_0}\right)dx$$

Reactor Model: One-Group

$$B_n^2 = \left(\frac{n\pi}{a_0}\right)^2 \quad \blacktriangleright \quad B_1^2 < B_3^2 < B_5^2 < \ldots$$

$$\lambda_n = v(\Sigma_a + DB_n^2 - \nu\Sigma_f) \quad \blacktriangleright \quad \lambda_1^2 < \lambda_3^2 < \lambda_5^2 < \ldots$$

$$\lambda_1 = v(\Sigma_a + DB_1^2 - \nu\Sigma_f)$$ Slowest decaying eigenvalue.

▼

$$\phi(x,t) \cong A_1 e^{-\lambda_1 t} \cos\left(\frac{\pi x}{a_0}\right) = A_1 e^{-\lambda_1 t} \cos B_1 x$$

Reactor Model: One-Group

For steady state $\lambda_1 = v(\Sigma_a + DB_1^2 - \nu\Sigma_f) = 0$

▼

Criticality $B_1^2 = B_g^2 = \dfrac{\nu\Sigma_f - \Sigma_a}{D} \equiv B_m^2$ $\lambda_1 = 0$

Super criticality $B_g^2 < B_m^2$ $LE \downarrow$ $\lambda_1 < 0$

Sub criticality $B_g^2 > B_m^2$ $LE \uparrow$ $\lambda_1 > 0$

Reactor Model: One-Group

- That was for the bare slab reactor.
- What about more general bare reactor models?

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla} \bullet D(\vec{r})\vec{\nabla}\phi(\vec{r},t)$$

- For steady state, homogeneous model:

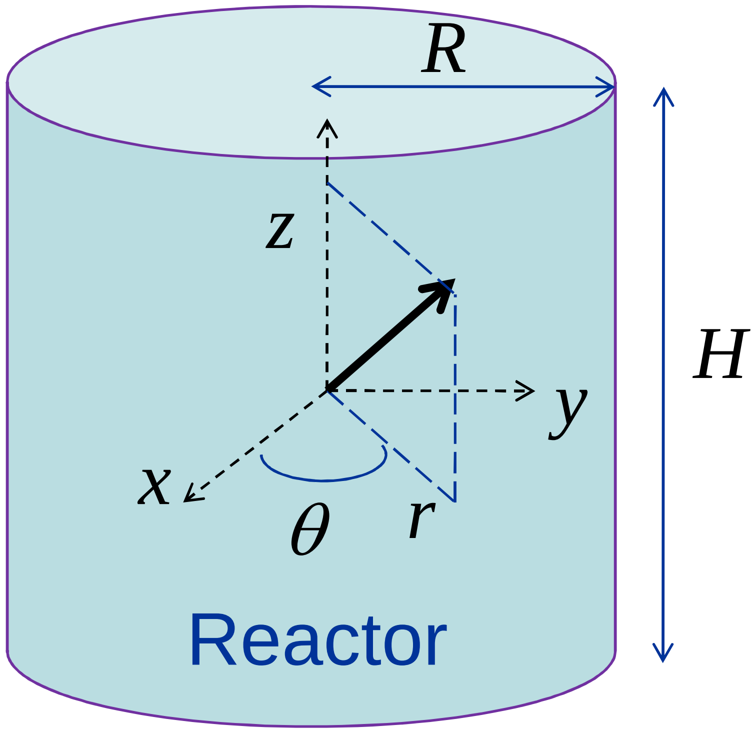
$$\nabla^2\phi(\vec{r},t) + \frac{\nu\Sigma_f - \Sigma_a}{D}\phi(\vec{r},t) = \nabla^2\phi(\vec{r},t) + \frac{k_\infty - 1}{L^2}\phi(\vec{r},t) = 0$$

- BC: ϕ*(extrapolated boundary) = 0.*

Reactor Model: One-Group

- R_0, H_0 are the extrapolated dimensions.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{dr}\right)+\frac{\partial^2 \phi}{dz^2}+B^2\phi=0$$



- BC's:

$$\phi(R_0, z)=0$$

$$\phi(r, \pm\tfrac{H_0}{2})=0$$

- Let

$$\phi(r,z)=\Re(r)Z(z)$$

Bessel *cos*

HW 26

- **Solve the problem and discuss criticality condition.**

Reactor Model: One-Group

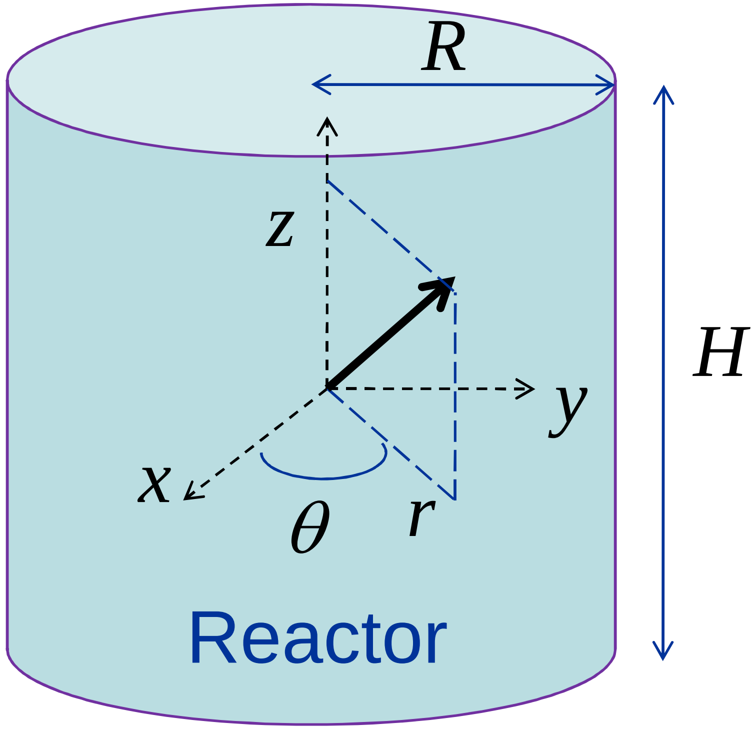
- Briefly, we go through HW 26.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{dr}\right)+\frac{\partial^2\phi}{dz^2}+B^2\phi=0$$

$$\phi(r,z)=\Re(r)Z(z)$$

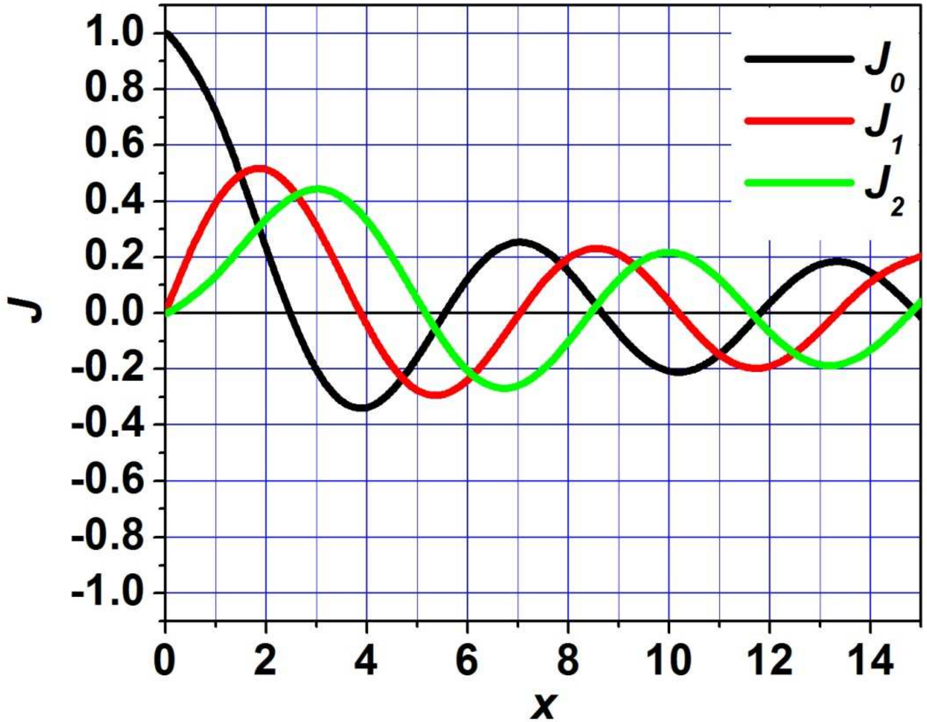
$$\frac{d^2Z}{dz^2}+\lambda^2Z=0\Rightarrow Z=\cos\lambda z=\cos\frac{\pi z}{H_0}$$

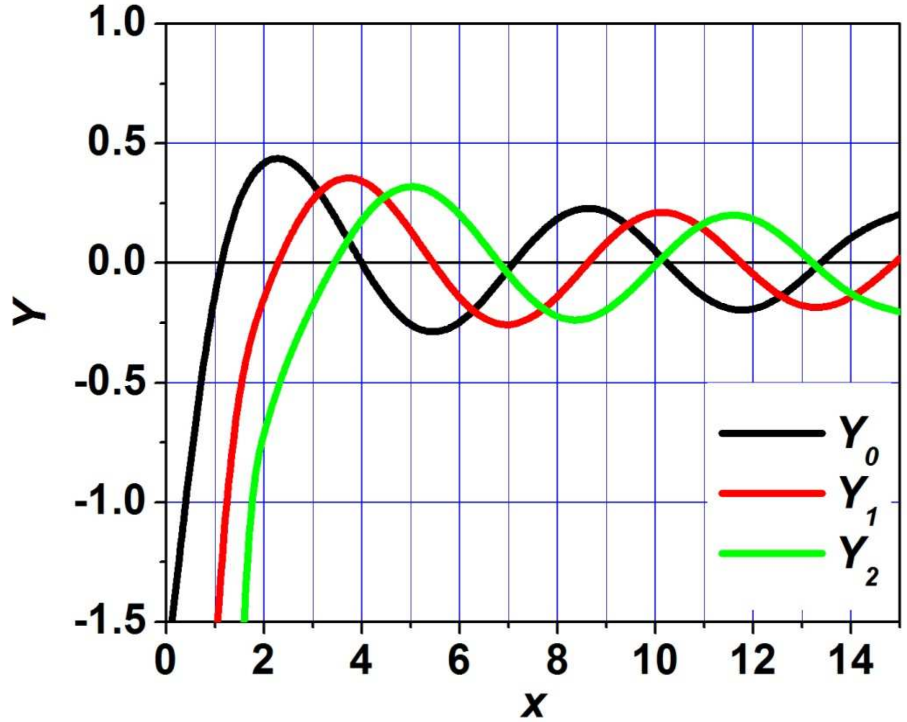
$$\frac{d}{dr}\left(r\frac{d\Re}{dr}\right)+\alpha^2\Re=0$$



$$\Re=AJ_0(\alpha r)+CY_0(\alpha r)$$

Reactor Model: One-Group



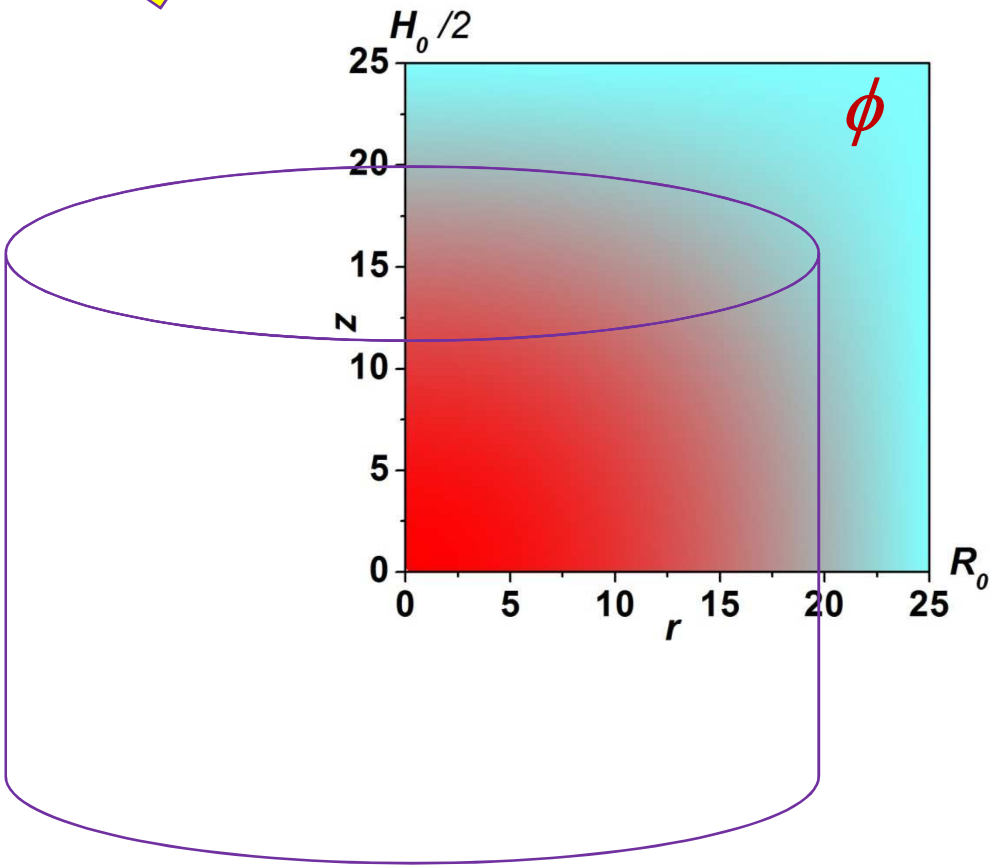


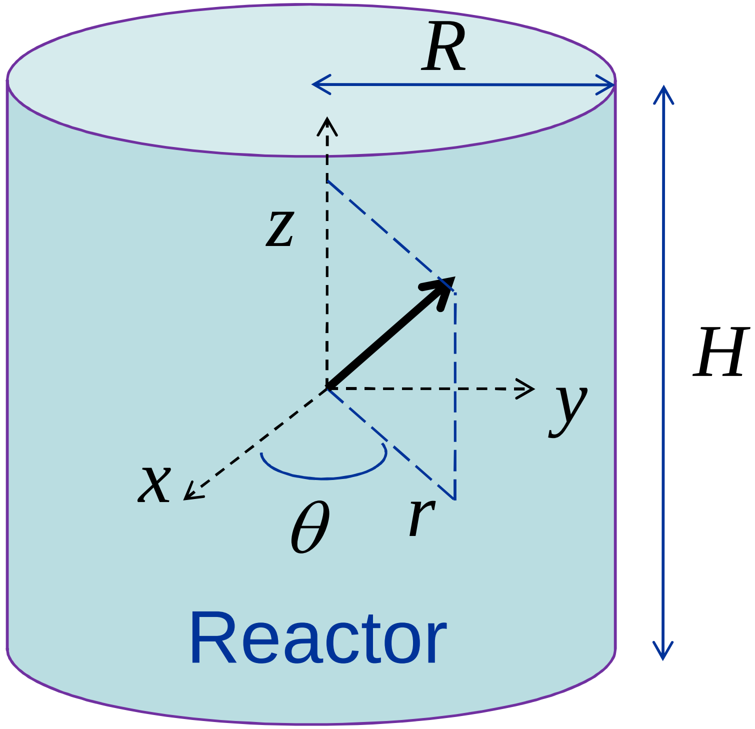
$$x \to 0 \Rightarrow Y_n(x) \to -\infty \Rightarrow C = 0$$

$$J_0(2.4048) = 0 \Rightarrow 2.4048 = \alpha R_0$$

Reactor Model: One-Group

$$\phi = A(P,\ldots)J_0\left(\frac{2.4048r}{R_0}\right)\cos\frac{\pi z}{H_0}$$





Criticality condition?

Do it.

Reactor Model: One-Group