More on One-Speed Diffusion

HW 20

Show that for a **critical homogeneous reactor**

$$P_{non-leak} = \frac{1}{B^2L^2+1} = \frac{\Sigma_a \phi}{\Sigma_a \phi - D\nabla^2\phi} = \frac{\Sigma_a \phi}{\Sigma_a \phi + B^2 D\phi}$$

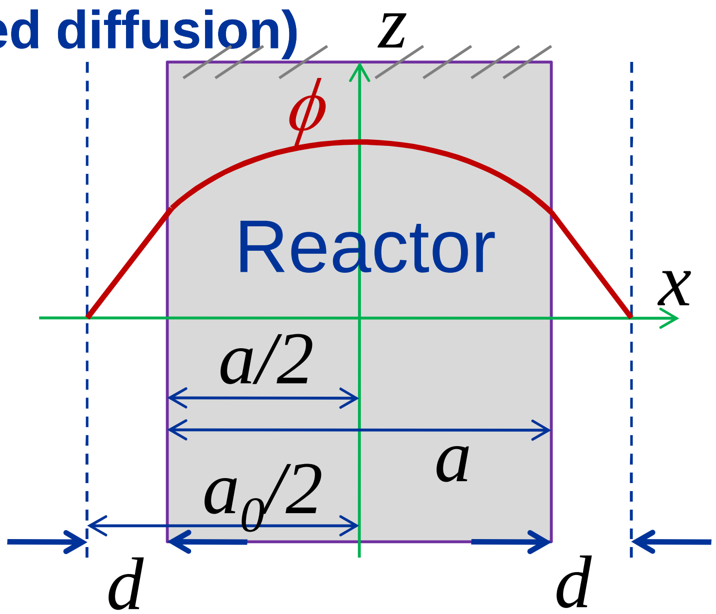
Infinite Bare Slab Reactor (one-speed diffusion)

- Vacuum beyond.
- Return current = 0.

d = linear extrapolation distance

= 0.71 λ_{tr} (for plane surfaces)

= 2.13 D.



More on One-Speed Diffusion

HW 21

For the infinite slab $\frac{d^2\phi}{dx^2} + B^2\phi = 0$. Show that the general solution

$$\phi(x) = A\cos Bx + C\sin Bx$$

with BC's

$$\phi(\pm\frac{a_0}{2}) = 0$$

$$\left.\frac{d\phi(x)}{dx}\right|_{x=0} = 0$$

Flux is symmetric about the origin.

$$\phi(x) = A\cos Bx \qquad A = \phi_0$$

$$\phi(\pm\frac{a_0}{2}) = A\cos B(\pm\frac{a_0}{2}) = 0 \Rightarrow B(\pm\frac{a_0}{2}) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

More on One-Speed Diffusion

HW 21 (continued)

$$B(\pm\frac{a_0}{2}) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$a_0 = \frac{\pi}{B}, \frac{3\pi}{B}, \frac{5\pi}{B}, \ldots$$

Fundamental mode, the only mode significant in critical reactors.

$$\phi(x) = \phi_0 \cos\frac{\pi}{a_0}x \qquad B^2 = \left(\frac{\pi}{a_0}\right)^2 \equiv \text{Geometrical Buckling}$$

For a critical reactor, the geometrical buckling is equal to the material buckling.

To achieve criticality →

$$\left(\frac{\pi}{a_0}\right)^2 = \frac{k_\infty - 1}{L^2}$$

More on One-Speed Diffusion

ϕ_0 ???

- To achieve criticality $\left(\frac{\pi}{a_0}\right)^2 = \frac{k_\infty - 1}{L^2}$
- But criticality at what power level??
- ϕ_0 can not be determined by the geometry alone.

$$\phi(x) = \phi_0(P,..,..)\cos\frac{\pi}{a_0}x$$

Do it.

More on One-Speed Diffusion

Spherical Bare Reactor (one-speed diffusion)

Cube $$\frac{6a^2}{a^3} > \frac{4\pi a^2}{\frac{4}{3}\pi a^3}$$ **Sphere**

Minimum leakage ▶ minimum fuel to achieve criticality.

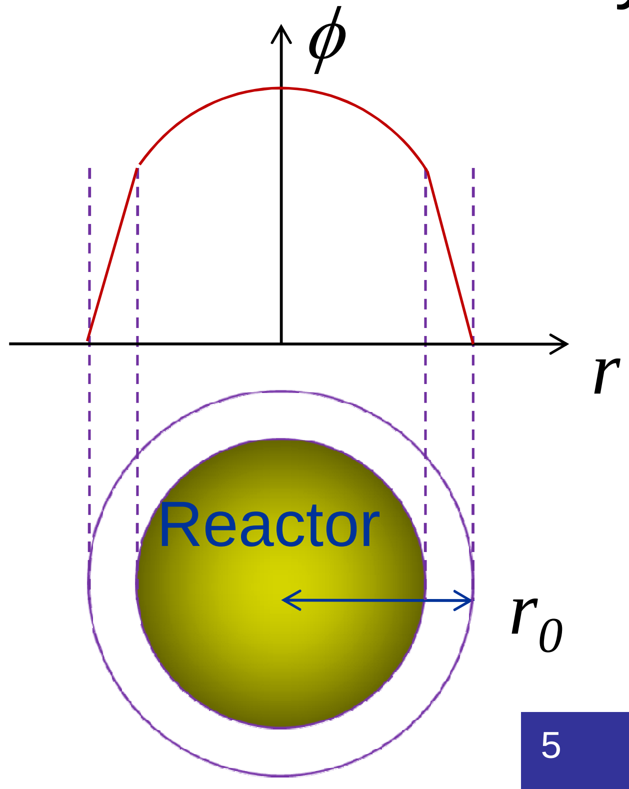
HW 22 $$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + B^2\phi = 0$$

$$\phi = \frac{A}{r}\cos Br + \frac{C}{r}\sin Br$$

▼

$$\phi = \frac{C}{r}\sin\frac{\pi r}{r_0}, \quad r_0 = \frac{\pi}{B}$$

Continue!

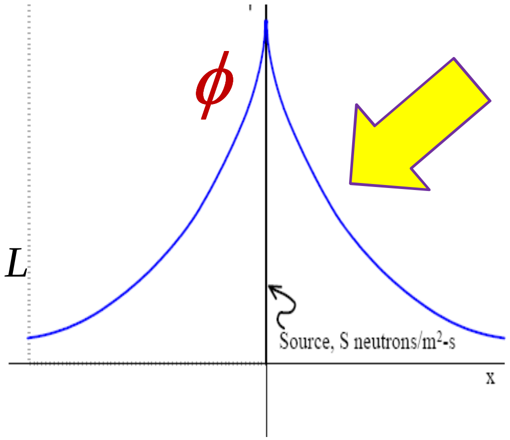


More on One-Speed Diffusion

HW 23

Infinite planer source in an infinite medium.

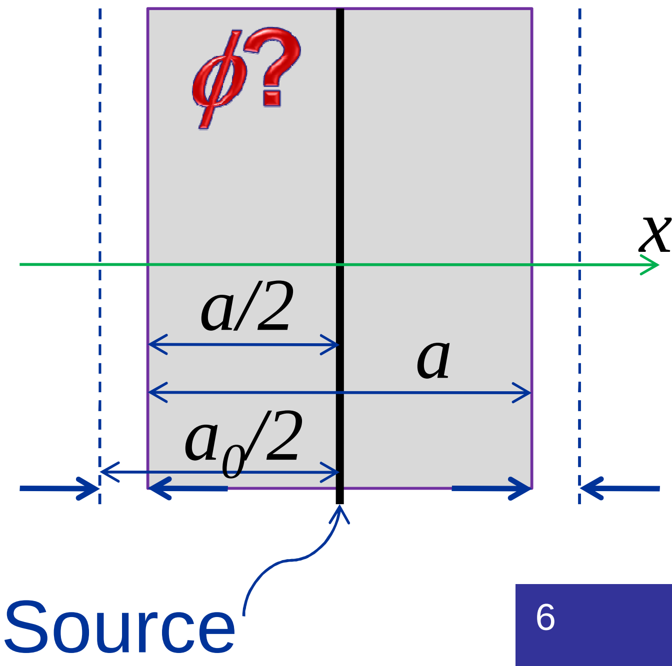
$$\frac{d^2\phi(x)}{dx^2} - \frac{1}{L^2}\phi = -\frac{S\delta(x)}{D} \quad \blacktriangleright \quad \phi(x) = \frac{SL}{2D} e^{-|x|/L}$$



HW 24

Infinite planer source in a finite medium.

$$\phi = \frac{SL}{2D} \frac{\sinh\left[\left(a_0 - 2|x|\right)/2L\right]}{\cosh(a_0/2L)}$$



More on One-Speed Diffusion

Infinite planer source in a multi-region medium.

$$\phi_1(\pm a/2) = \phi_2(\pm a/2)$$

$$D_1 \frac{d\phi_1}{dx}\bigg|_{x=\pm a/2} = D_2 \frac{d\phi_2}{dx}\bigg|_{x=\pm a/2}$$

$$+ more \qquad BC$$

Project 2

Infinite Finite Infinite