Fick's Law

- The exact interpretation of neutron transport in heterogeneous domains is so complex.
- Assumptions and approximations.
- Simplified approaches.
- Simplified but accurate enough to give an **estimate** of the **average characteristics** of **neutron population**.
- Numerical solutions.
- Monte Carlo techniques.

MCNP

Fick's Law

Assumptions:

1. The medium is infinite.
2. The medium is uniform $\sum not \sum(\vec{r})$
3. There are no neutron sources in the medium.
4. Scattering is isotropic in the lab coordinate system.
5. The neutron flux is a slowly varying function of position.
6. The neutron flux is not a function of time.

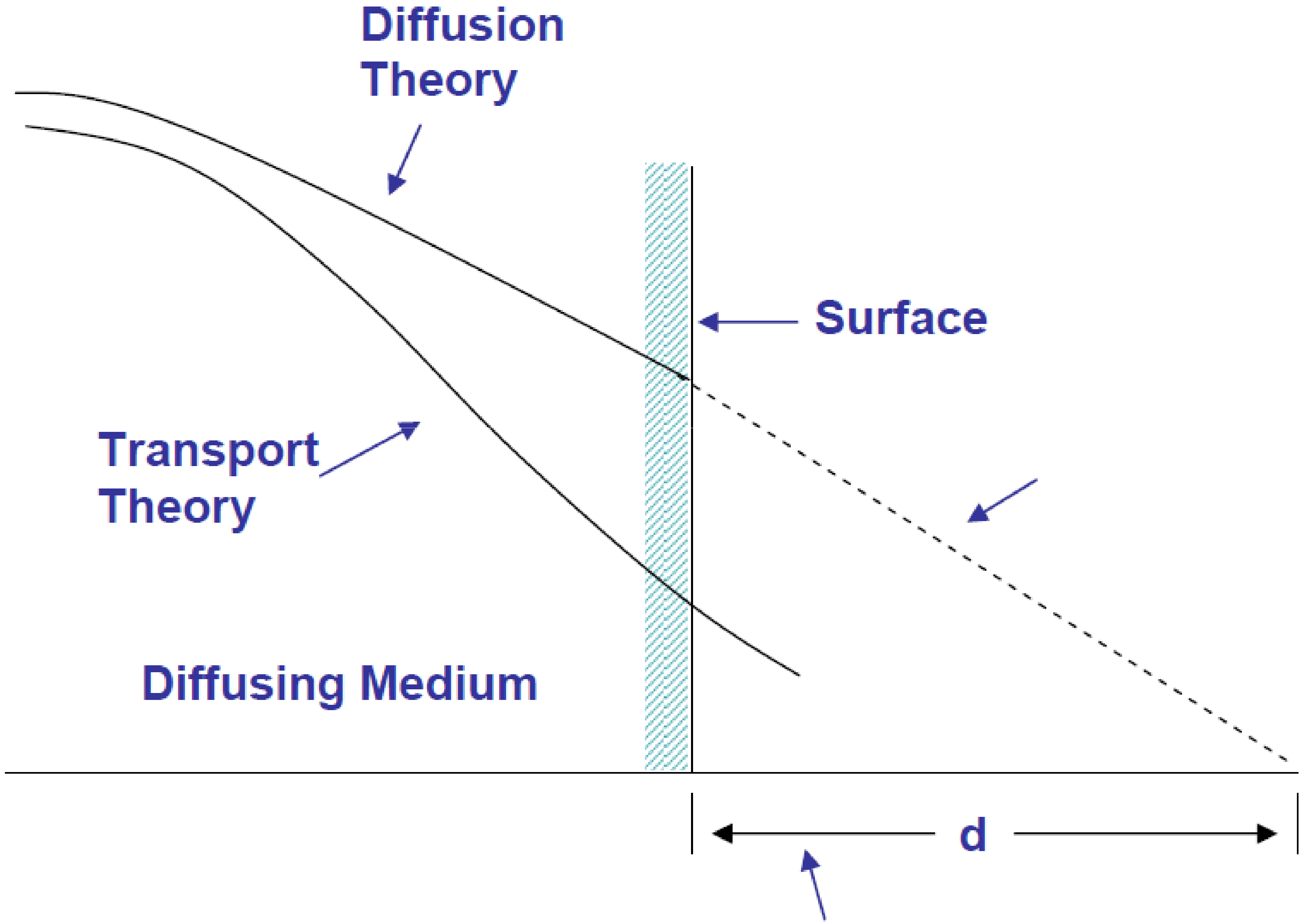
Restrictive! **Applicability??**

http://www.iop.org/EJ/article/0143-0807/26/5/023/ejp5_5_023.pdf

Fick’s Law

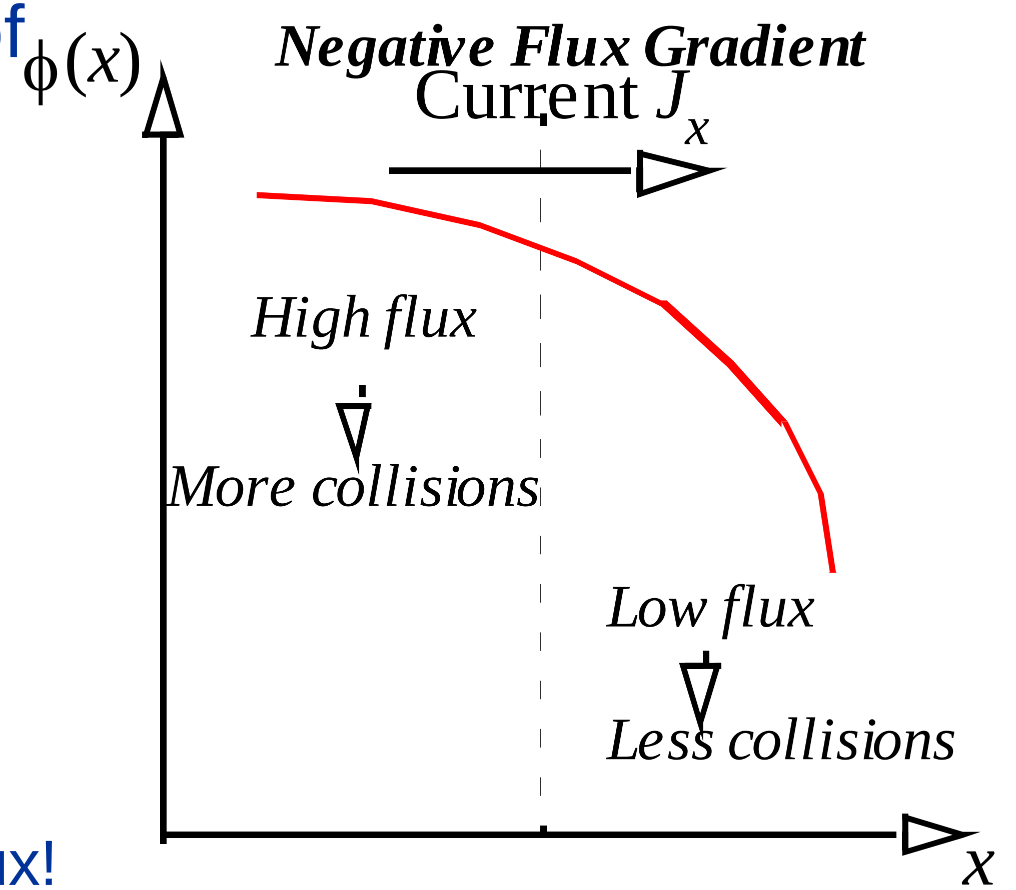
Lamarsh puts it more bluntly:
“Fick’s Law is invalid:
a) in a medium that strongly absorbs neutrons;
b) within three mean free paths of either a neutron source or the surface of a material; and
c) when neutron scattering is strongly anisotropic.”

Fick's Law


Diffusion
Theory
Surface
Transport
Theory
Diffusing Medium
d

Fick's Law

- Diffusion: random walk of an ensemble of particles from region of high "concentration" to region of small "concentration".
- Flow is proportional to the negative gradient of the "concentration".

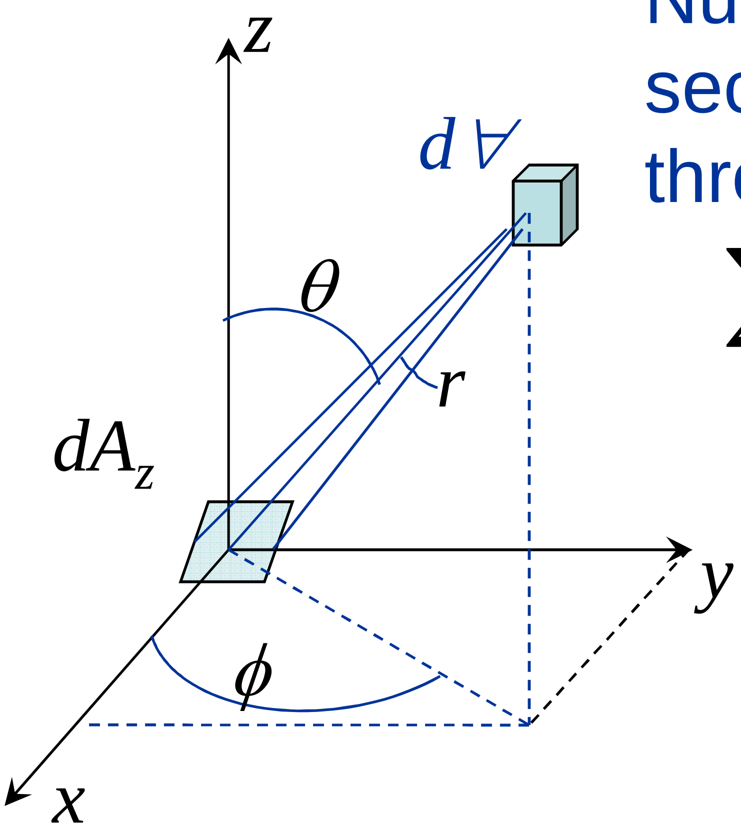


$$J_x = -D\frac{\partial \phi}{\partial x}$$

Recall:

- From larger flux to smaller flux!
- Neutrons are not pushed!
- More scattering in one direction than in the other.

Fick's Law



Number of neutrons **scattered** per second from $d\forall$ at $\mathbf{r}$ and going through dA_z

$$\Sigma_s \phi(\vec{r}) \frac{\cos\theta dA_z}{4\pi r^2} e^{-\Sigma_t r} d\forall$$

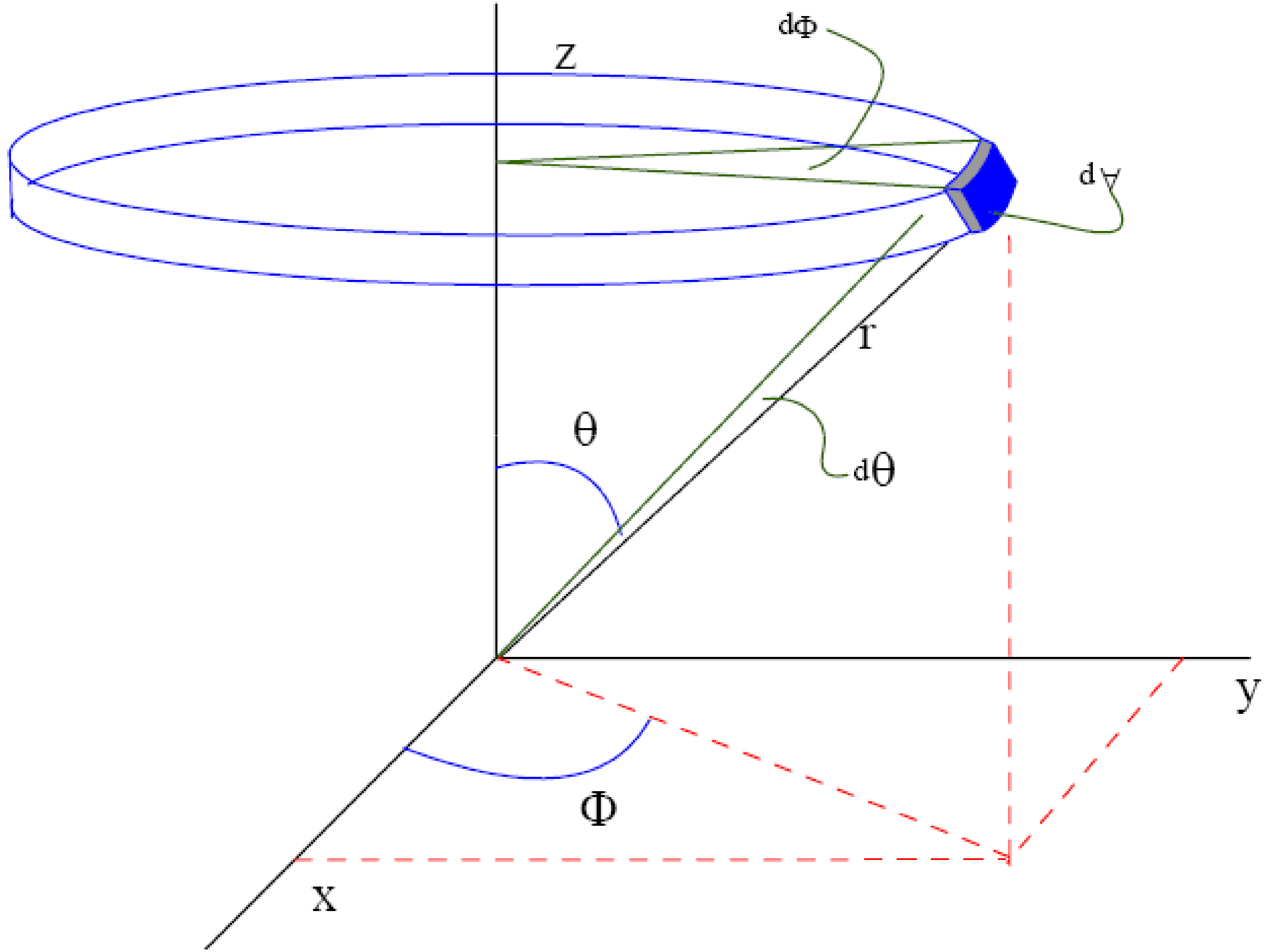
Σ_s *not* $\Sigma_s(\vec{r})$

Slowly varying

Removed *en route* (assuming no buildup)

Isotropic

Fick's Law



Fick's Law

HW 14

$$J_z^- dA_z = \frac{\Sigma_s dA_z}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} \phi(\vec{r}) e^{-\Sigma_t r} \left[\cos\theta \sin\theta dr d\theta d\phi\right]$$

$$J_z^+ dA_z = ?$$

and show that $J_z = J_z^+ - J_z^- = -\left(\frac{\Sigma_s}{3\Sigma_t^2}\right)\left(\frac{\partial \phi}{\partial z}\right)_0$

$D \approx \frac{1}{3\Sigma_s}$?

Fick's law

and generalize $J = -D\vec{\nabla}\phi$

Diffusion coefficient $D = \frac{\Sigma_s}{3\Sigma_t^2}$ → Total removal

The current density is proportional to the negative of the gradient of the neutron flux.

Fick's Law

Validity:

1. The medium is infinite. Integration over all space.
   $e^{-\Sigma_t r}$ ▶ after few mean free paths ▶ 0 ▶
   corrections at the surface are still required.
2. The medium is uniform. $\Sigma_s \; not \; \Sigma_s(\vec{r})$
   $\Sigma_s(\vec{r})$ ▶ ϕ and Σ are functions of space ▶ re-derivation of Fick's law? ▶ locally larger Σ_s ▶ extra ***J*** cancelled by $e^{-\Sigma_t r} = e^{-(\Sigma_a + \Sigma_s) r}$ iff ??? **HW 15**
   Note: assumption 5 is also violated!
3. There are no neutron sources in the medium.
   Again, sources are few mean free paths away and corrections otherwise.

Fick's Law

4. Scattering is isotropic in the lab. coordinate system.

If $\bar{\mu} = \overline{\cos(\theta)} = \frac{2}{3A} \neq 0$ ▶ reevaluate *D*. **HW 16**

$$D = \frac{1}{3(\Sigma_t - \Sigma_s \bar{\mu})} = \frac{1}{3\Sigma_{tr}} = \frac{\lambda_{tr}}{3}$$

Weekly absorbing $\Sigma_t = \Sigma_s$.

For "practical" moderators: $\lambda_{tr} \cong \frac{\lambda_s}{1-\bar{\mu}}$

5. The flux is a slowly varying function of position.

$\Sigma_a \uparrow$ ▶ variation in $\phi \uparrow$.

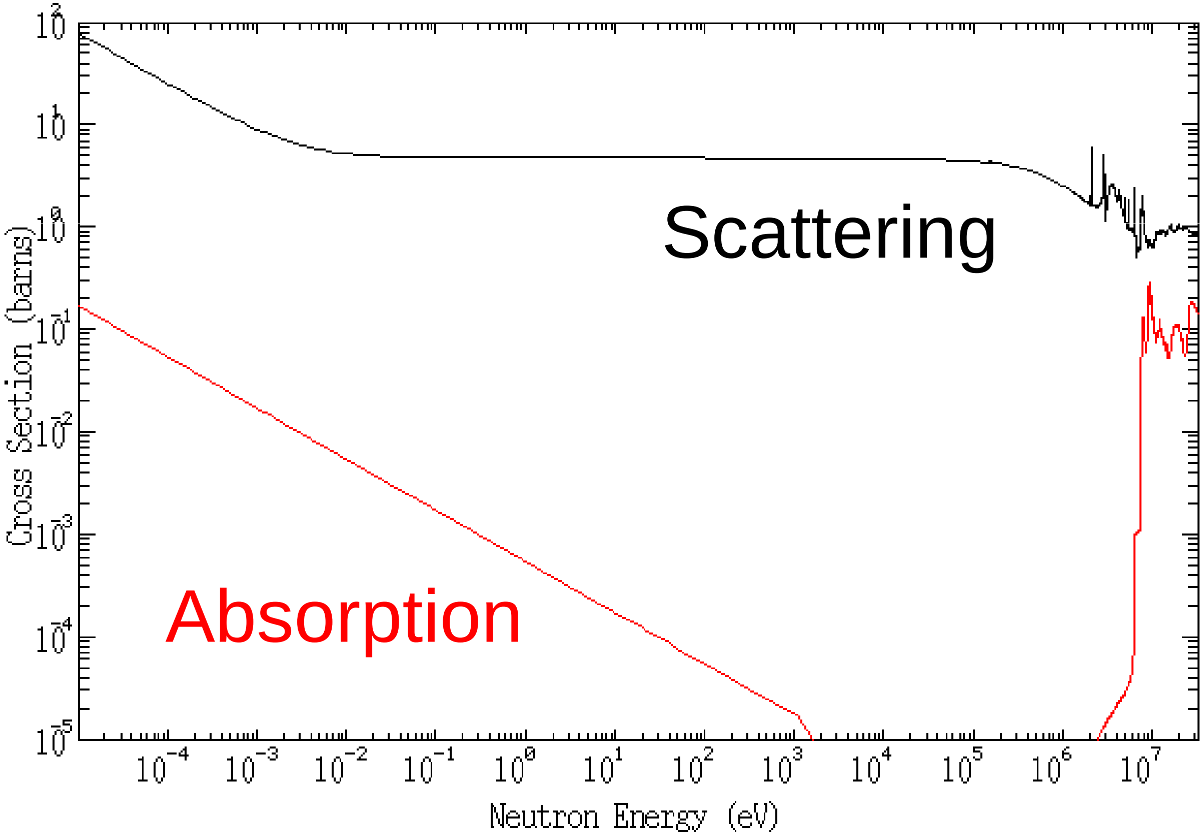
$$\frac{\partial^2 \phi}{\partial r^2}(\vec{r}) \quad \textbf{?}$$

Fick's Law

HW 17

Estimate the diffusion coefficient of graphite at 1 eV.

The scattering cross section of carbon at 1 eV is 4.8 b.



Other materials?

Fick's Law

6. The neutron flux is not a function of time.

Time needed for a thermal neutron to traverse 3 mean free paths ~ 1 x 10^{-3} s (How?).

If flux changes by 10% per second!!!!!!

$$\left.\frac{\Delta\phi}{\phi}\right|_{1ms} = \frac{\Delta\phi/\phi}{1s} 1ms = 0.1x10^{-3} = 1x10^{-4}$$

Very small fractional change during the time needed for the neutron to travel this "significant" distance.

$$J \cong -D\vec{\nabla}\phi$$

Back to the Continuity Equation

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) - \vec{\nabla}\bullet\vec{J}(\vec{r},t)$$

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla}\bullet D\vec{\nabla}\phi(\vec{r},t)$$

The Diffusion Equation

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \vec{\nabla}\bullet D\vec{\nabla}\phi(\vec{r},t)$$

If D is independent of **r** (uniform medium)

The Diffusion Equation

Laplacian

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + D\nabla^2\phi(\vec{r},t)$$

The Steady State Diffusion Equation or scalar Helmholtz equation.

$$0 = S(\vec{r}) - \Sigma_a(\vec{r})\phi(\vec{r}) + D\nabla^2\phi(\vec{r})$$

Non-multiplying medium (and steady state)

$$0 = -\Sigma_a(\vec{r})\phi(\vec{r}) + D\nabla^2\phi(\vec{r})$$

Buckling equation.

Steady State Diffusion Equation

$$0 = S(\vec{r}) - \Sigma_a(\vec{r})\phi(\vec{r}) + D\nabla^2\phi(\vec{r})$$

Define $$L^2 = \frac{D}{\Sigma_a}$$

$L \equiv$ Diffusion Length

$L^2 \equiv$ Diffusion Area

Moderation Length

$$\nabla^2\phi - \frac{1}{L^2}\phi = -\frac{S}{D}$$

$$\nabla^2\phi - \frac{1}{L^2}\phi = 0$$

Non-multiplying medium

Boundary Conditions

- Solve DE ▶ get ϕ.
- Solution must satisfy BC's.
- Solution should be real and non-negative.

Steady State Diffusion Equation

One-speed neutron diffusion in infinite medium

Point source

$$\nabla^2\phi(\vec{r}) - \frac{1}{L^2}\phi(\vec{r}) = 0$$

HW 18

$$\frac{d^2}{dr^2}\phi(r) + \frac{2}{r}\frac{d}{dr}\phi(r) - \frac{1}{L^2}\phi(r) = 0$$

General solution

$$\phi = A\frac{e^{-r/L}}{r} + C\frac{e^{r/L}}{r}$$

A, *C* determined from BC's.

Steady State Diffusion Equation

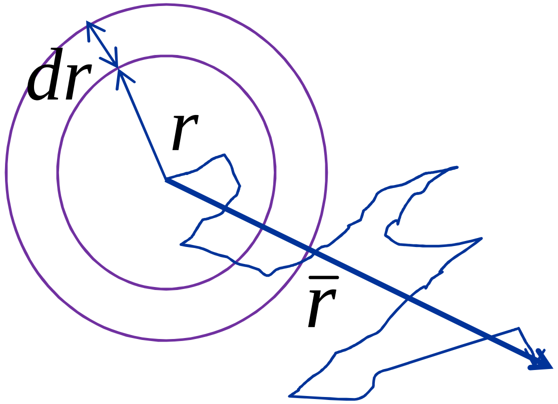
BC $r \to \infty$ ▶ $\phi \to 0$ ▶ $C = 0.$

HW 18 (continued)

$$\phi = A\frac{e^{-r/L}}{r}$$

Show that $A = \dfrac{S}{4\pi D}$ ▶ $$\phi = \frac{S}{4\pi D}\frac{e^{-r/L}}{r}$$ $L^2 = \dfrac{D}{\Sigma_a}$

$4\pi r^2 dr\,\Sigma_a\,\phi$ neutrons per second absorbed in the ring.



Show that $$\overline{r^2} = 6L^2$$

Steady State Diffusion Equation

Scalar flux, vector current.

HW 19

Study example 5.3 and solve problem 5.8 in Lamarsh.

Multiple Point Sources?

Steady State Diffusion Equation

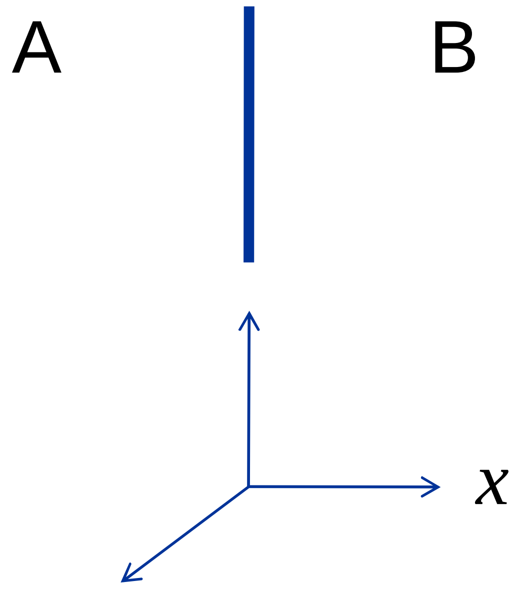
One-speed neutron diffusion in a finite medium

- At the interface

$$\phi_A = \phi_B$$

$$J_A = J_B \Rightarrow -D_A \frac{d\phi_A}{dx} = -D_B \frac{d\phi_B}{dx}$$

- What if *A* or *B* is a vacuum?
- Linear extrapolation distance.



More realistic multiplying medium

One-speed neutron diffusion in a multiplying medium

The reactor core is a finite multiplying medium.

- Neutron flux?
- Reaction rates?
- Power distribution in the reactor core?

Recall:

- **Critical (or steady-state):**

Number of neutrons produced by fission = number of neutrons lost by:

absorption

and

leakage

$$k_\infty = \frac{\text{neutron production rate}\,(S)}{\text{neutron absorption rate}\,(A)}$$

$$k_{eff} = \frac{\text{neutron production rate}\,(S)}{\text{neutron absorption rate}\,(A) + \text{neutron leakage rate}\,(LE)}$$

More realistic multiplying medium

Things to be used later…!

$$\frac{k_{eff}}{k_\infty} = \frac{A}{A+LE} = P_{non-leak}$$ non - leakage probability

$LE \propto SA$ surface area

$S \propto V$ *Volume*

$$\frac{LE}{S} \propto \frac{SA}{V} \propto \frac{a^2}{a^3} = \frac{1}{a}$$

Recall:

For a critical reactor:

$K_{eff} = 1$

$K_\infty > 1$

Steady state homogeneous reactor

$$0 = \Sigma_a k_\infty \phi(\vec{r}) - \Sigma_a \phi(\vec{r}) + D\nabla^2 \phi(\vec{r})$$

multiplying medium

$$\nabla^2 \phi(\vec{r}) + B^2 \phi(\vec{r}) = 0 \qquad B^2 \equiv \frac{k_\infty - 1}{L^2}$$

Material buckling

More realistic multiplying medium

$$\nabla^2 \phi(\vec{r}) + B^2 \phi(\vec{r}) = 0$$

$$B^2 = -\frac{\nabla^2 \phi(\vec{r})}{\phi(\vec{r})}$$

- The buckling is a measure of extent to which the flux curves or "buckles."
- For a slab reactor, the buckling goes to zero as "*a*" goes to infinity. There would be no buckling or curvature in a reactor of infinite width.
- Buckling can be used to infer leakage. The greater the curvature, the more leakage would be expected.

More on One-Speed Diffusion

HW 20

Show that for a **critical homogeneous reactor**

$$P_{non-leak} = \frac{1}{B^2L^2+1} = \frac{\Sigma_a \phi}{\Sigma_a \phi - D\nabla^2\phi} = \frac{\Sigma_a \phi}{\Sigma_a \phi + B^2 D\phi}$$

Infinite Bare Slab Reactor (one-speed diffusion)

- Vacuum beyond.
- Return current = 0.

d = linear extrapolation distance

= 0.71 λ_{tr} (for plane surfaces)

= 2.13 D.