Fusion Reactions

Self Study

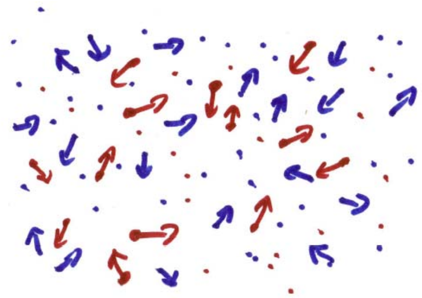
Carefully read Krane 14.1, 14.2 and 14.3.

Read also 14.4.

Fusion Reactions

- Neutron-induced fission ▶ No Coulomb barrier.
- Charged particle-induced fusion ▶ Coulomb barrier.

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$



- Thermonuclear reactions.
- At room temperature ▶ kT = 0.025 eV.
- Practically, keV available energy but much higher Coulomb barrier.
- What is the temperature required to classically overcome the barrier for a D-D reaction.
- Penetration probability ▶ much lower temperatures.

Fusion Reactions

- We formulated the cross section when we considered charged particle reactions.

$$\sigma(E) = \frac{1}{E} e^{-2\pi\eta} S(E)$$

Fusion Reactions

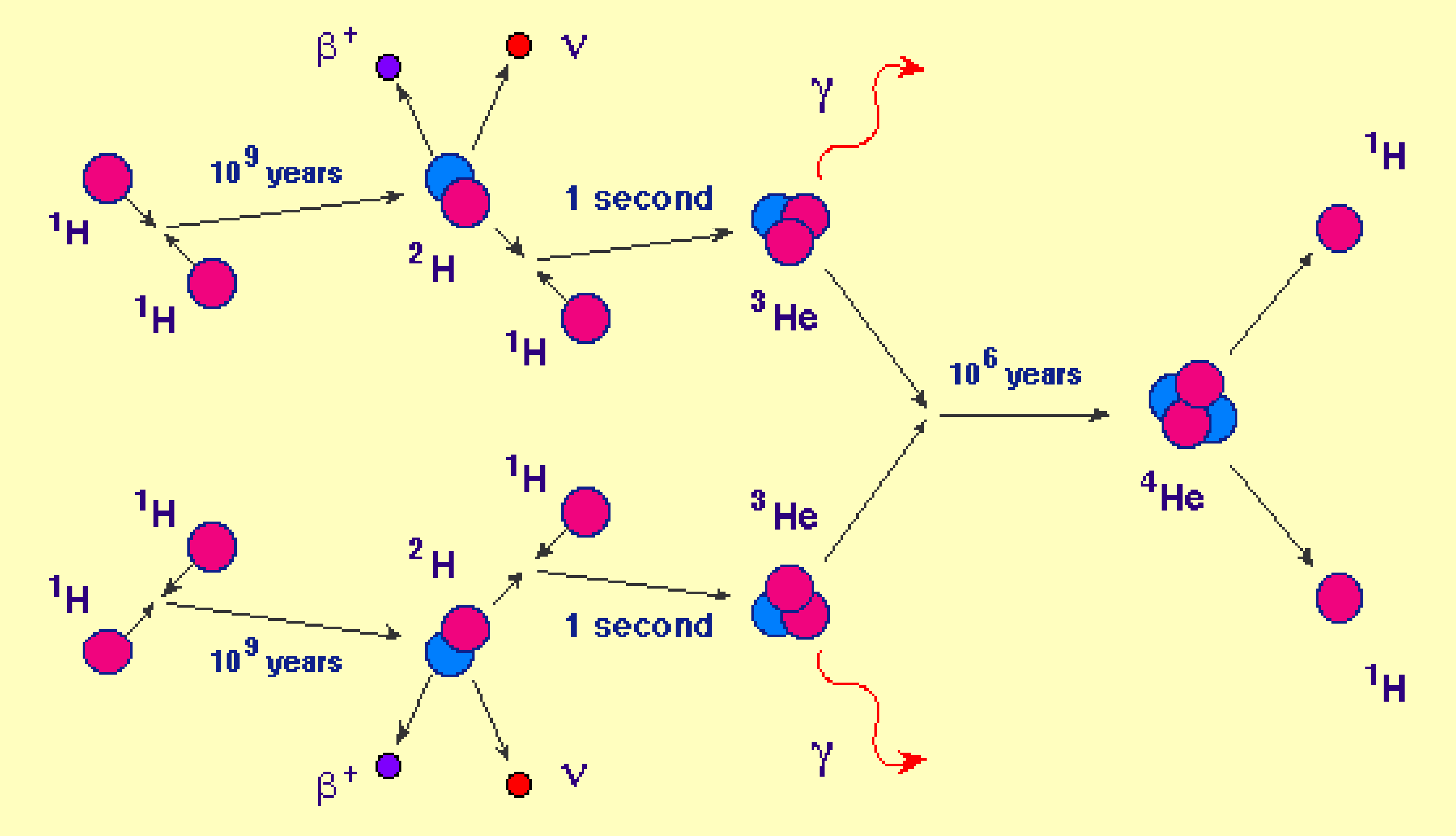
- Show that

$$\tfrac{1}{2} m_b v_b^2 = \frac{Q}{1 + m_b / m_Y}$$

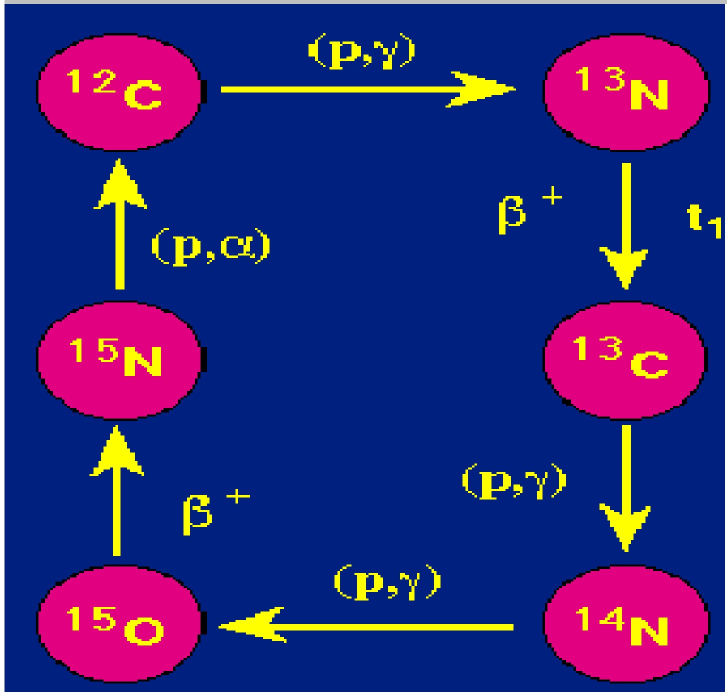
$$\tfrac{1}{2} m_Y v_Y^2 = \frac{Q}{1 + m_Y / m_b}$$

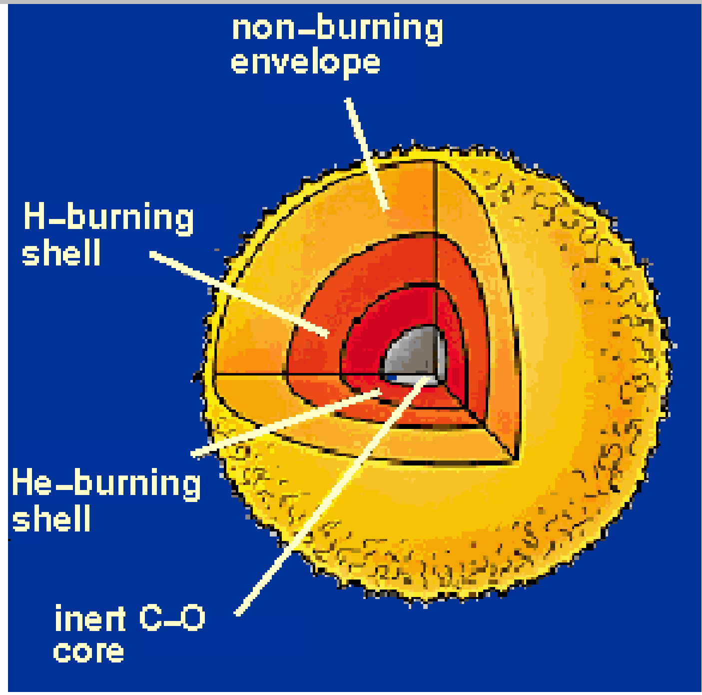
$$\frac{\tfrac{1}{2} m_b v_b^2}{\tfrac{1}{2} m_Y v_Y^2} = \frac{m_Y}{m_b}$$

Fusion Reactions



Fusion Reactions





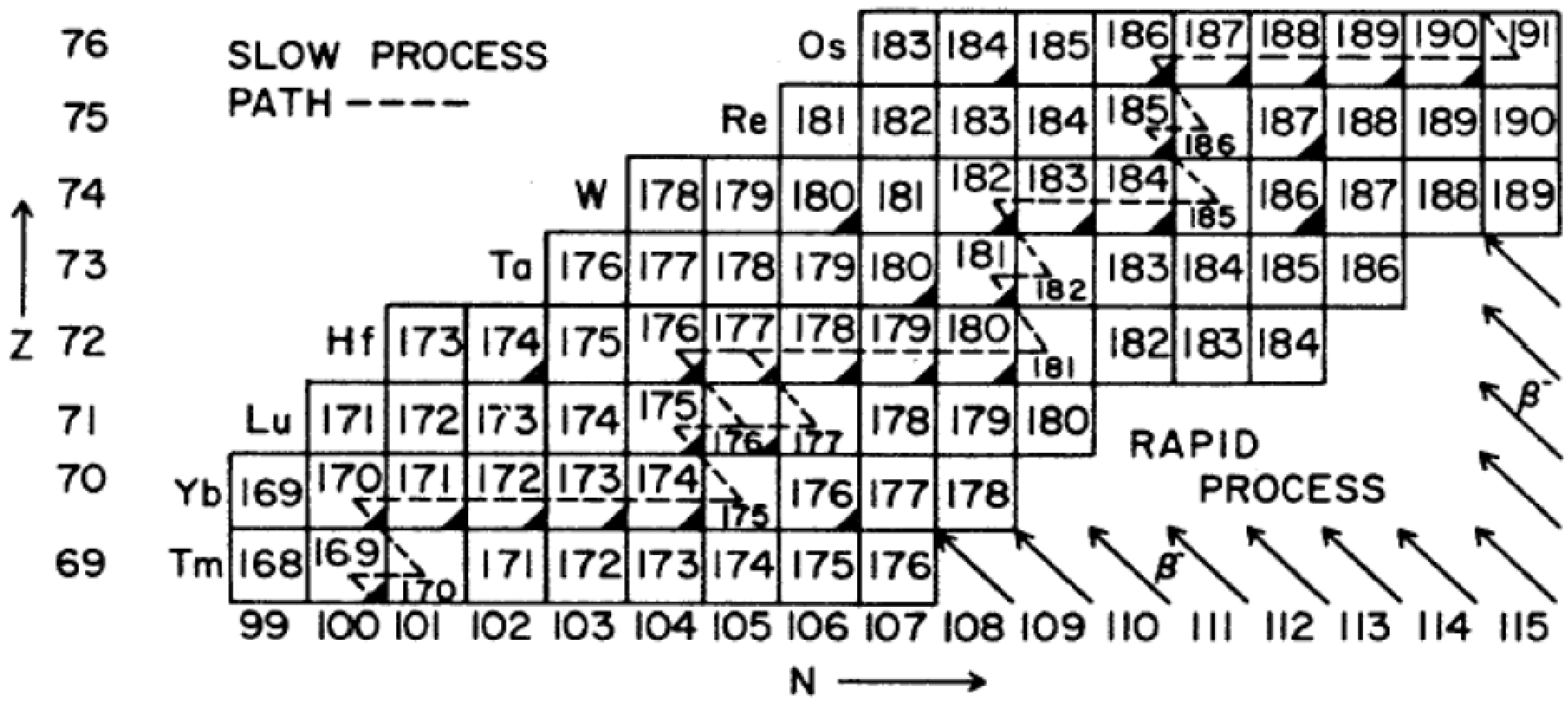
Experimental challenges!

Neutron Sources for the s-Process

$^{13}C(\alpha,n)$ $\quad$ $^{12}C(p,\gamma)^{13}N(\beta^+)^{13}C(\alpha,n)^{16}O$

$^{22}Ne(\alpha,n)$ $\quad$ $^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O(\alpha,\gamma)^{22}Ne(\alpha,n)$

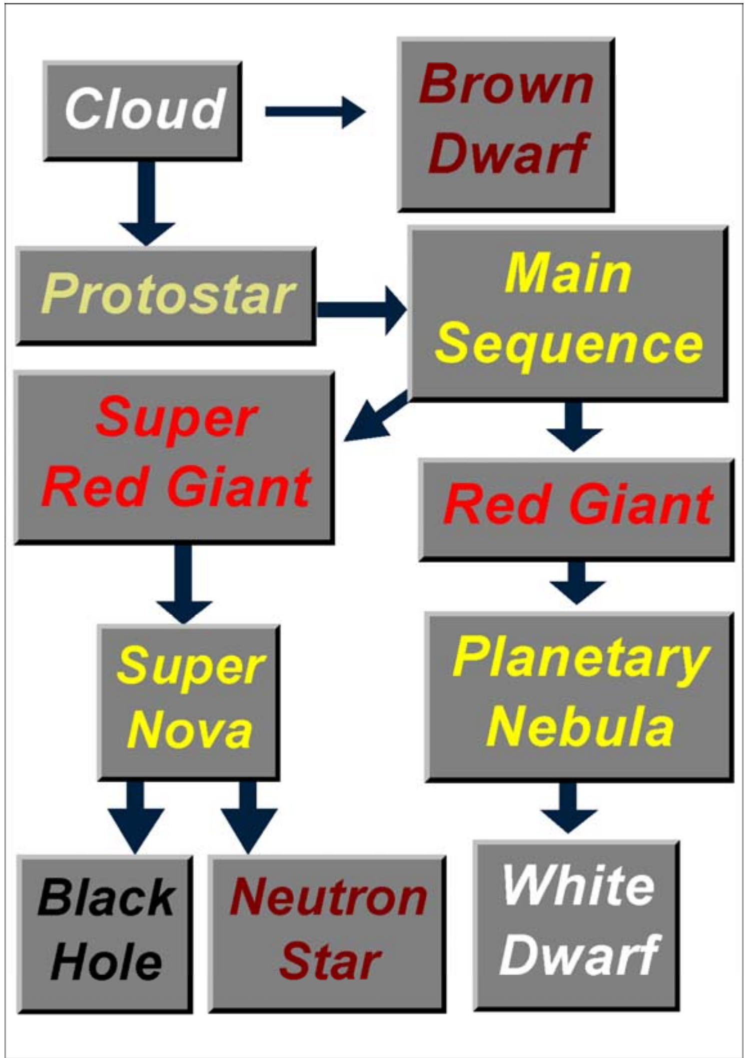
Fusion Reactions

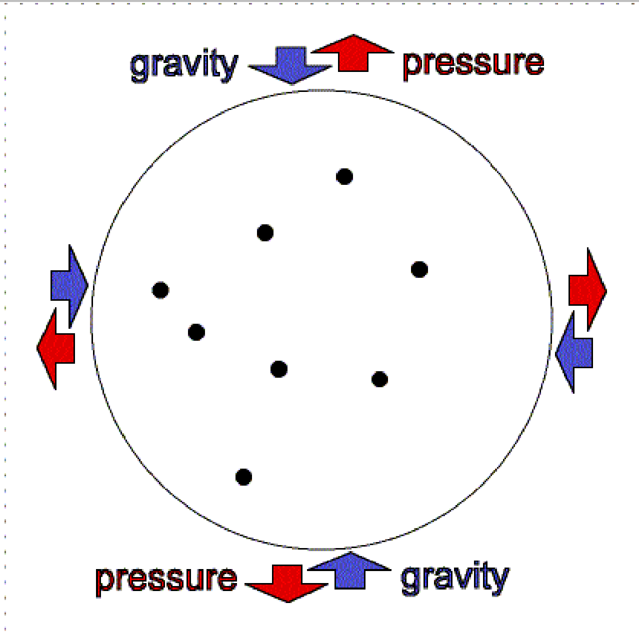


Neutron capture but neutron source is a fusion reaction.

Fusion Reactions

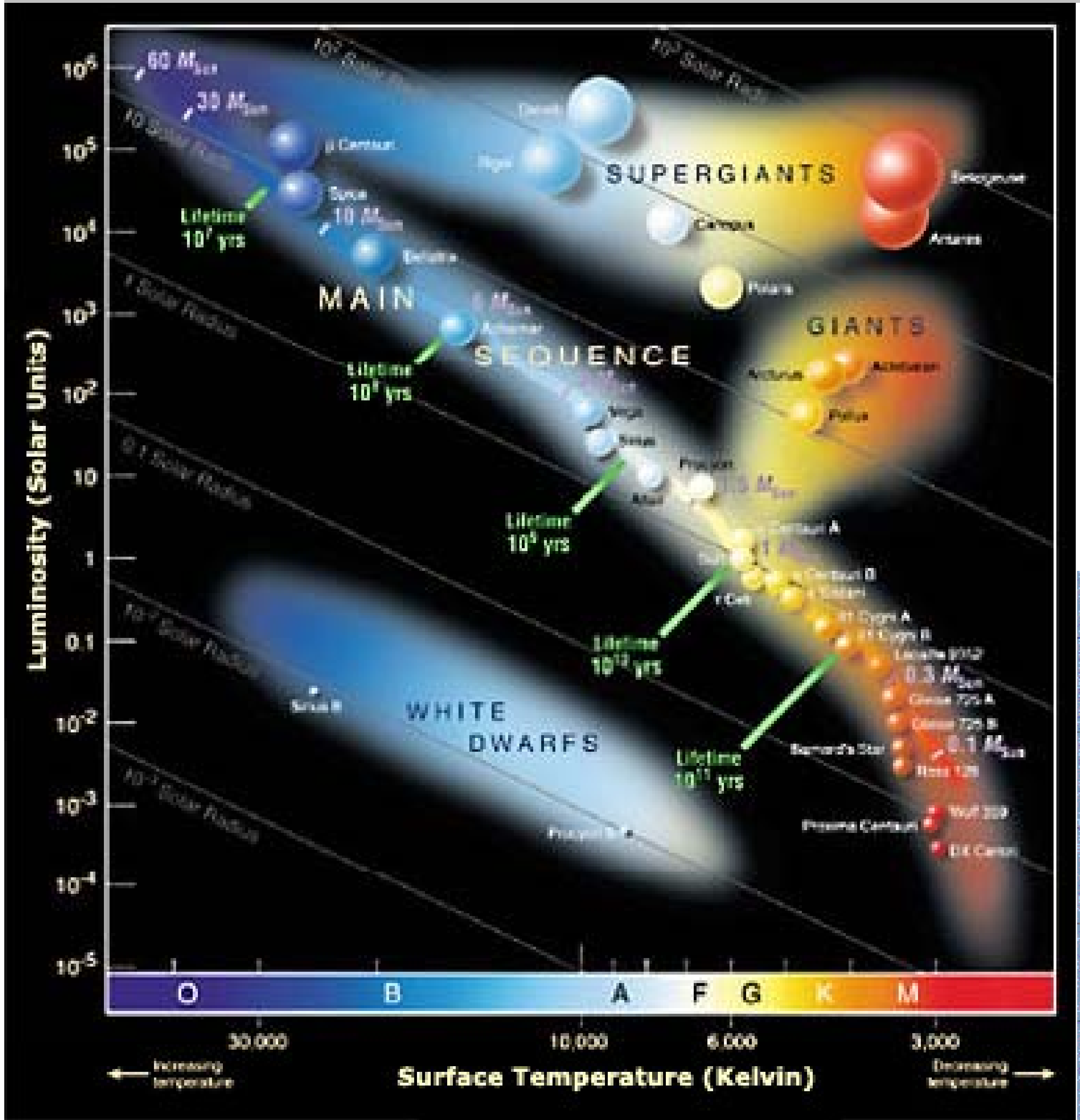
Stellar Evolution (Overview)

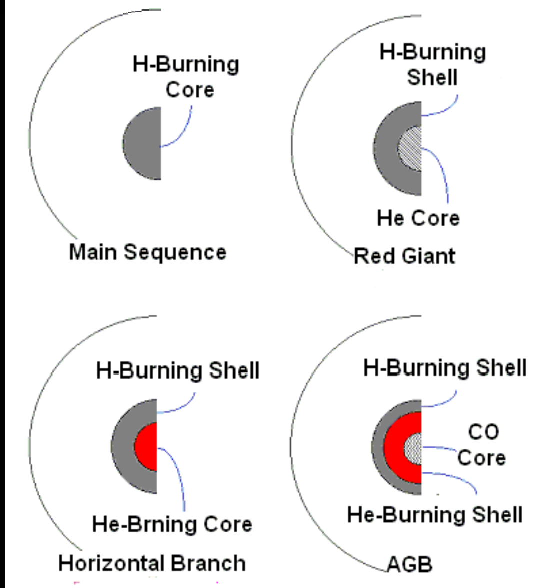


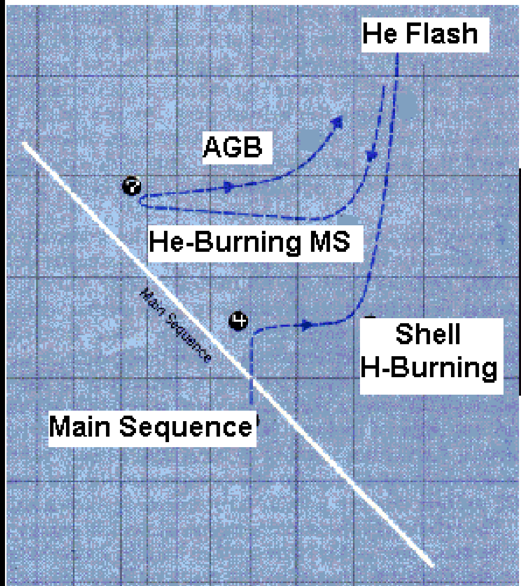


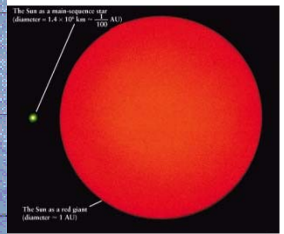
- **Hydrostatic equilibrium.**
- **Mass distribution.**
- **Chemical composition.**
- **Energy production and transport.**
- **Temperature (and pressure) gradient.**
- **Thermonuclear reaction rates are important input.**

Fusion Reactions

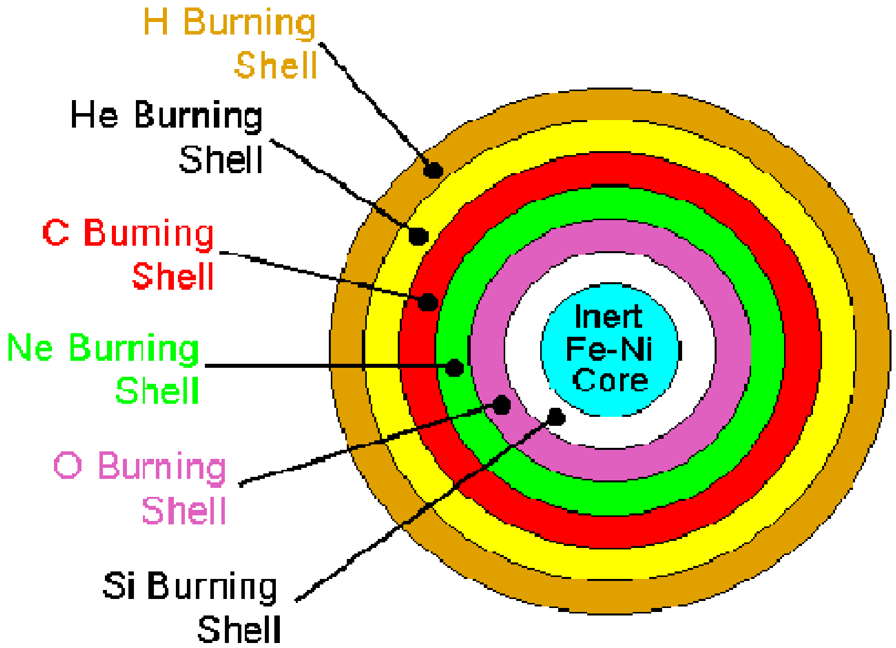






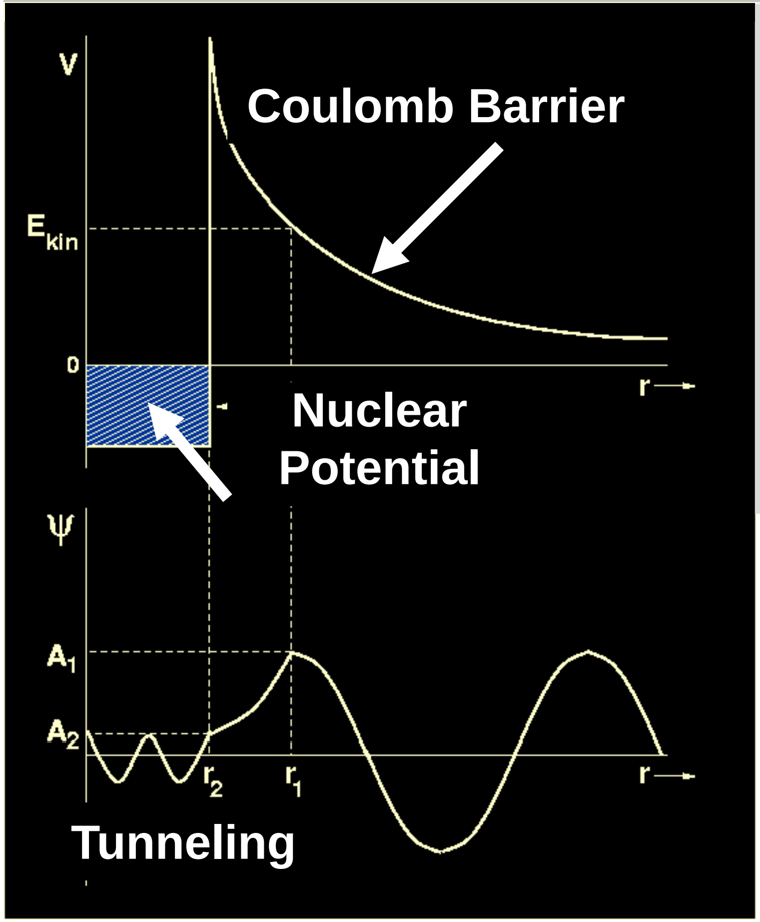


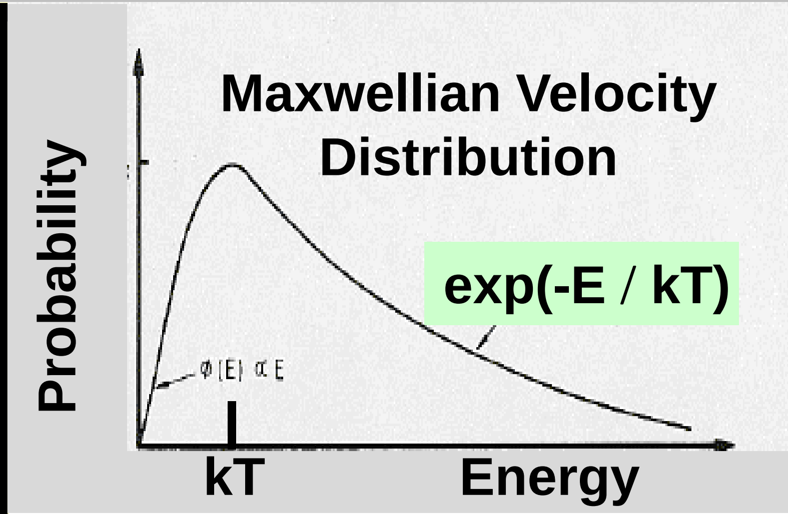
Fusion Reactions



- **Evolution depends on mass.**
- ***s*-process has 2 components.**
- **Neutron sources are alpha-induced reactions.**

Fusion Reactions





- Kinetic Energy?
- $V = Z_1 Z_2 e^2 / r$
  $E_C \approx Z_1 Z_2$ MeV
- Sun: $E_C = 1$ MeV
  $kT = 1$ keV

$\exp(-E / kT)$
$= e^{-1000} = 10^{-434}$!!!
$^{18}O(\alpha,\gamma)$ 10^{-700}

- Tunneling
  $e^{-2\pi\eta}$

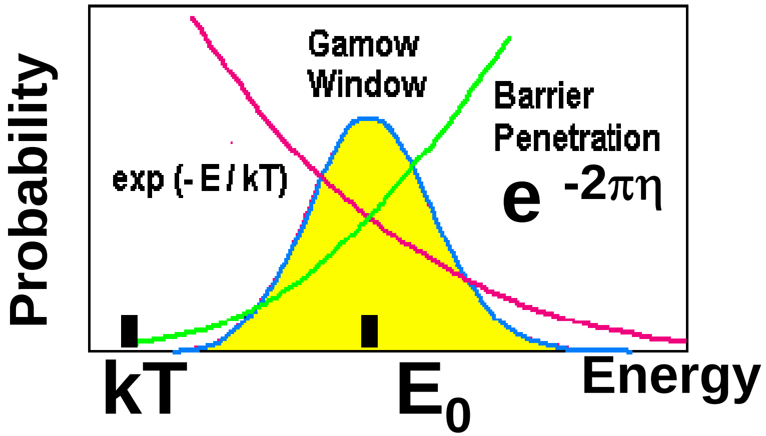
$\eta \propto Z_1 Z_2 (m/E)^{0.5}$

- Well separated phases of nuclear burning.

Thermonuclear Reactions

Fusion Reactions

Thermonuclear Reaction Rate



- **The reaction probability per particle pair, averaged over the velocity distribution**

$$< \sigma v > = (\frac{8}{\pi\mu})^{1/2} \frac{1}{(kT)^{3/2}} \int \sigma(E) E \exp(-\frac{E}{kT}) dE$$

$$= (\frac{8}{\pi\mu})^{1/2} \frac{1}{(kT)^{3/2}} \int S(E) \exp(-2\pi\eta - \frac{E}{kT}) dE$$

- **The integrand has appreciable values only around the Gamow energy.**
- **He-burning T = 0.1 – 0.3 billion degree.**
  kT = 10 – 30 keV
  300 – 600 keV is Astrophysically important for the $^{18}O(\alpha,\gamma)$ reaction.

Fusion Reactions

Thermonuclear Resonant Reactions

$$\langle \sigma v \rangle \propto E_R e^{-E_R/kT} \int_0^\infty \sigma(E) \mathrm{d}E$$

$$\int \sigma(E) \mathrm{d}E \propto \bar{\lambda}^2 \omega \frac{\Gamma_a \Gamma_b}{\Gamma}$$

$$\omega = \frac{2I+1}{(2I_1+1)(2I_2+1)}$$

Resonance Strength

$\omega\gamma$

Low energy ?

Barrier penetration

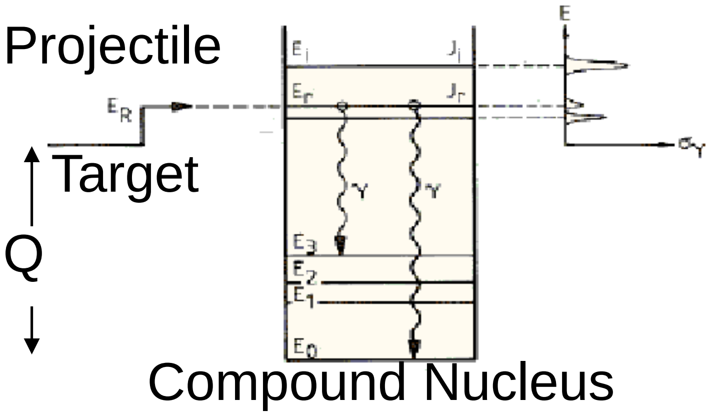
$\Gamma_\alpha << \Gamma_\gamma$

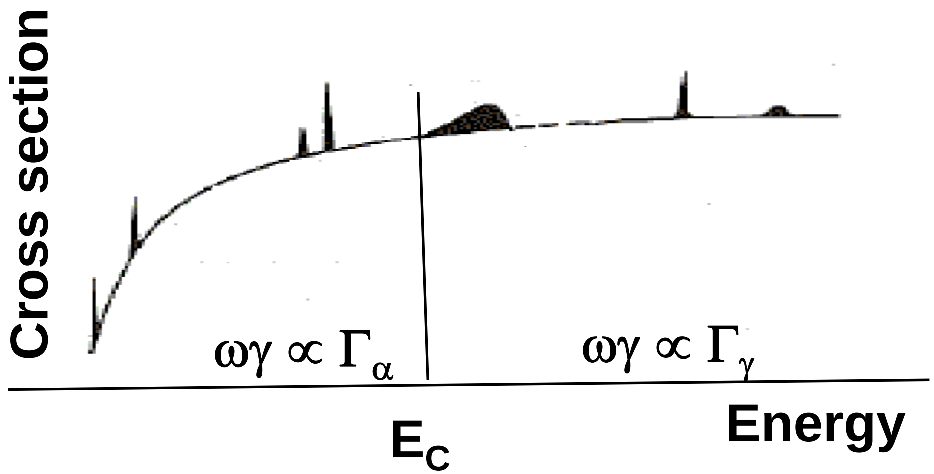
$\omega\gamma \approx \omega\, \Gamma_\alpha$

Yield $\approx \lambda^2 \omega\gamma / \varepsilon$

$\approx \lambda^2 \omega\, \Gamma_\alpha / \varepsilon$

Background ??





Fusion Reactions in Stellar He-Burning

He-Burning Reactions

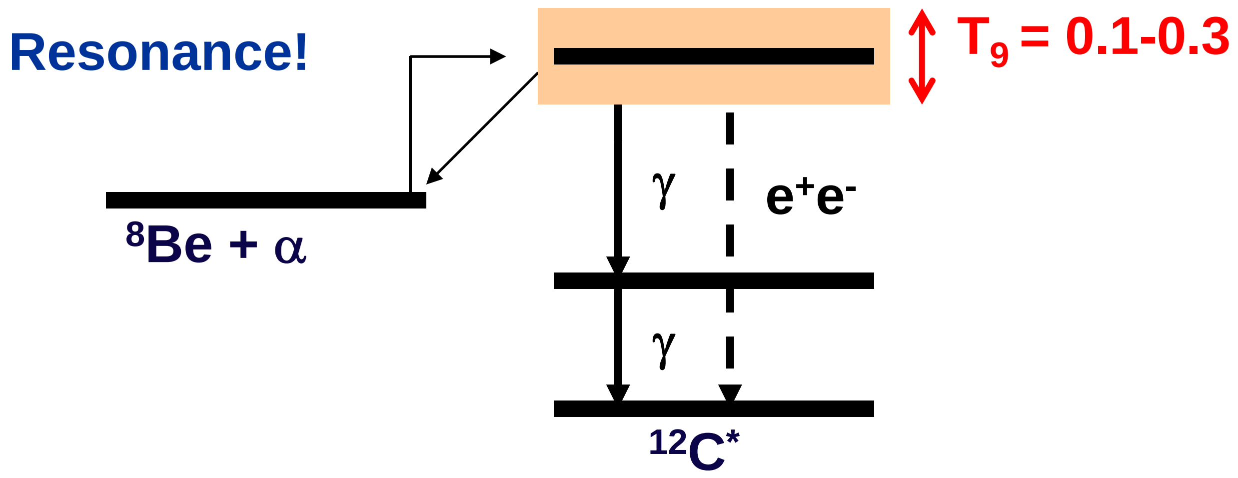
Triple-alpha Process

- **Stability gap at A = 5, 8**
- **Bottleneck at temperatures as "low" as 10 million K in MS stars.**
- **At higher temperatures of 100 million K in Red Giants**

$$^{4}He + {}^{4}He \leftrightarrow {}^{8}Be$$

with high enough "rate" so that

$$^{8}Be + {}^{4}He \rightarrow {}^{12}C^{*}$$



$$<\sigma v> \propto \omega\gamma \exp(-E_R/kT)$$

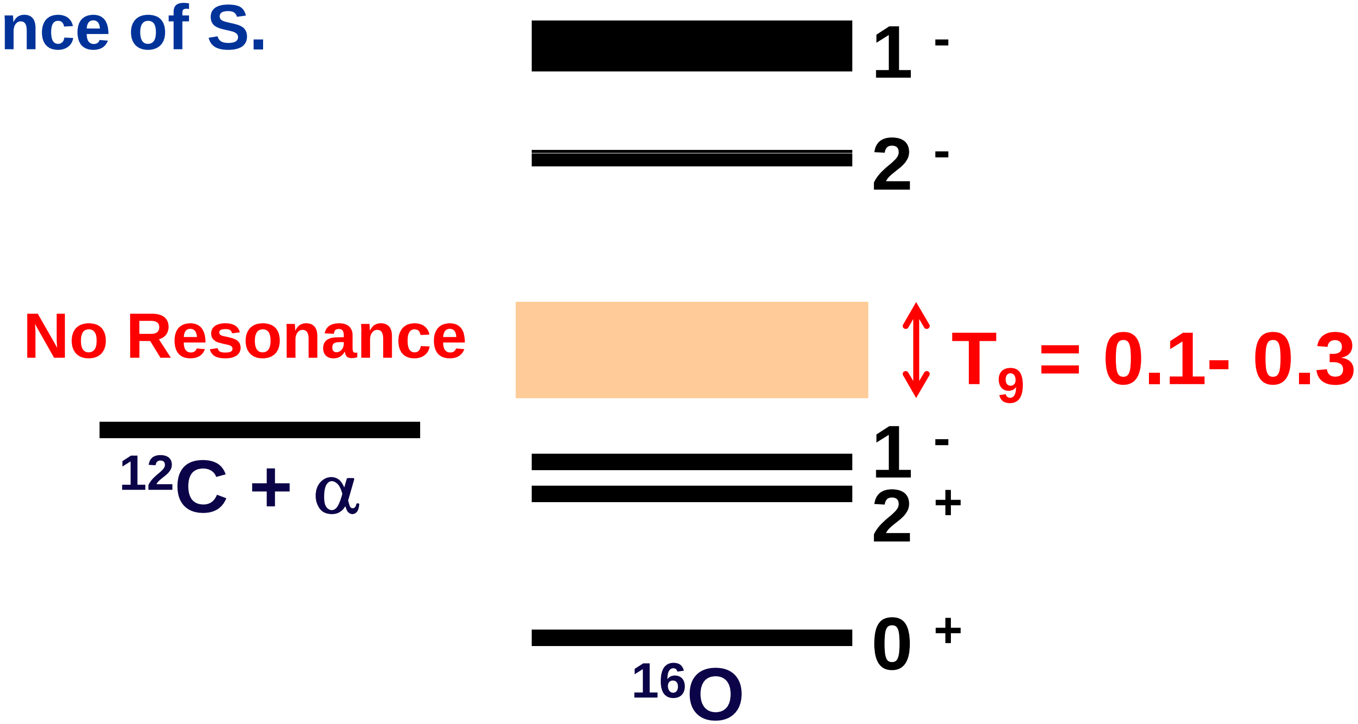
Fusion Reactions in Stellar He-Burning

Survival of ^{12}C in Red Giants

- **No resonance near Gamow Window.**
- **Direct capture and tails of broad and subthreshold resonances.**

$$\sigma = S\, e^{-2\pi\eta} / E$$

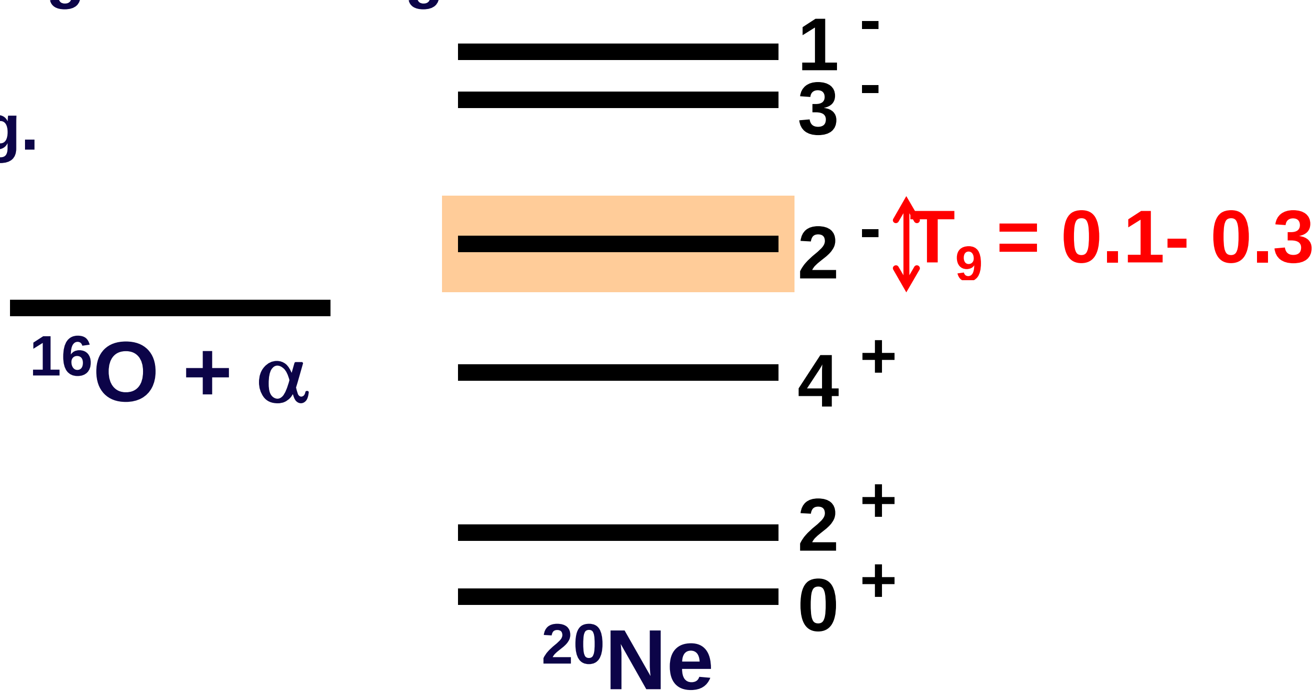
- **Energy dependence of S.**



Fusion Reactions in Stellar He-Burning

Blocking Reaction

- $^{16}O \rightarrow {}^{20}Ne \rightarrow {}^{24}Mg \rightarrow {}^{28}Si$ …. ??
- **Increase in Coulomb Barrier.**
- **Properties of the resonances.**
- **Only direct capture could contribute, blocking He-burning nucleosynthesis.**
- **Advanced burning.**



Fusion Reactions in Stellar He-Burning

What if?

- **the mass of ^{8}Be had not been close to the mass of 2 alpha particles…?**
- **there were no enhancing resonant state in ^{12}C….?**
- **there were no parity conservation law……?**