

Nuclear Parity

- $\psi(r) \rightarrow \psi(-r)$ Even.
- $\psi(r) \rightarrow -\psi(-r)$ odd.
- For a nucleon ψ is either of even ($\pi = +$) or odd ($\pi = -$) parity.
- For the nucleus $\pi = \pi_1 \pi_2 \pi_3 \dots \pi_A$.
- Individually not possible.
- Overall π can be determined experimentally.
- Overall I^π for a nucleus (nuclear state).
- Transitions and **multipolarity** of transitions (γ -emission).

Electromagnetic moments

- Electromagnetic interaction ► information about nuclear structure.
- Charge ► electric; current ► magnetic.
- Electromagnetic multipole moments.

Field $\propto 1/r^2$ (zeroth, $L=0$) electric monopole moment.

$1/r^3$ (first, $L=1$) electric dipole moment.

$1/r^4$ (second, $L=2$) quadrupole moment.

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$1/r^2$ magnetic monopole (questionable....!).

Magnetic Dipole (familiar).

Higher order magnetic moments.

Electromagnetic moments

- Expectation value of the moment. $\int \psi^* \mathcal{O} \psi dv$
- Each multipole moment has a parity, determined by the behavior of the multipole operator when $r \rightarrow -r$.
- Parity of ψ does not change the integrand.
- Electric moments: parity $(-1)^L$.
- Magnetic moments: parity $(-1)^{L+1}$.
- Odd parity \blacktriangleright vanish.

Vanishing moments

electric dipole.
magnetic quadrupole.
electric octupole.

.....

Electromagnetic moments

- Electric monopole: net charge Ze .
- Magnetic dipole:

$$\mu = iA$$

$$\mu = \frac{e}{T} A ; \quad \mu = \frac{e}{2\pi r/v} \pi r^2 ; \quad \mu = \frac{evr}{2}$$

$$\mu = \frac{emvr}{2m} ; \quad \mu = \frac{epr}{2m} ; \quad \mu = \frac{e}{2m} L$$

- g-factors.

Nuclear Magnetic Moment

Remember, for electrons

Revise: Torque on a current loop.

$$\mu_B = \frac{e\hbar}{2m_e} = 9.2740154 \times 10^{-24} \text{ J / T} = 5.7883826 \times 10^{-5} \text{ eV / T}$$

Bohr magneton

Orbital

$$\mu_L = -g_L \frac{e}{2m_e} L \qquad \mu_{Lz} = -g_L \frac{e\hbar}{2m_e} m_\ell = -m_\ell \mu_B$$

Gyromagnetic ratio (g-factor) $g_L = 1$

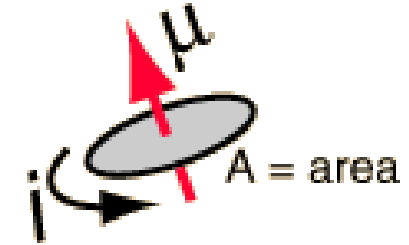
Spin

$$\mu_S = -g_S \frac{e}{2m_e} S \qquad \mu_{Sz} = -g_S \frac{e\hbar}{2m_e} m_s = -2m_s \mu_B = \pm \mu_B$$

$$g_S = 2.0023 \approx 2$$

Z component ?? Experiment, applied magnetic field.

Nuclear Magnetic Moment



For Nuclei

$$\mu = g \frac{e}{2m_p} I \quad \mu_z = g \frac{e\hbar}{2m_p} m_I = g\mu_N m_I$$

Nuclear magneton $\mu_N = 5.05084 \times 10^{-27} \text{ J/T} = 3.15245 \times 10^{-8} \text{ eV/T}$

For free protons and neutrons

$$\mu_z = \frac{1}{2} g\mu_N$$

Proton: $g = 5.5856912 \pm 0.0000022 \quad \sim 3.6 \updownarrow$

Neutron: $g = -3.8260837 \pm 0.0000018 \quad \sim 3.8 \updownarrow$

The proton g-factor is far from the $g_s = 2$ for the electron, and even the uncharged neutron has a sizable magnetic moment!!!

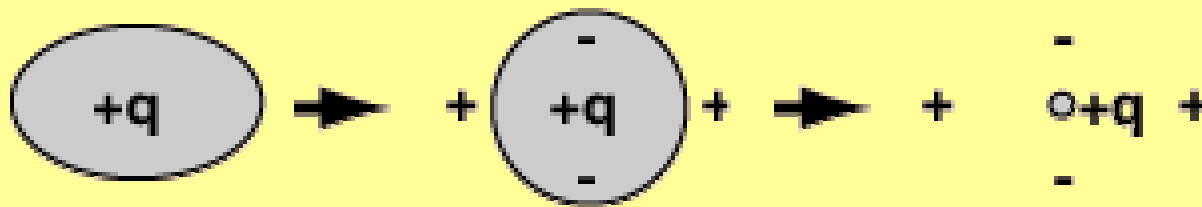
► **Internal structure (quarks).**

Nuclear Magnetic Moment

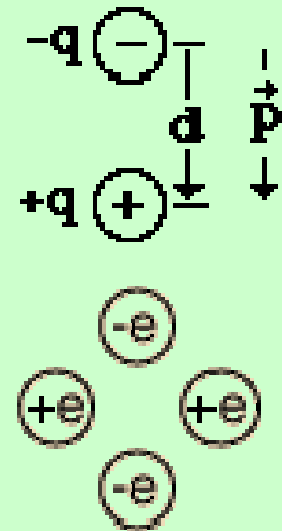
Nuclide	Nuclear spin	Magnetic moment μ (in μ_N)
n	1/2	-1.9130418
p	1/2	+2.7928456
^2H (D)	1	+0.8574376
^{17}O	5/2	-1.89279
^{57}Fe	1/2	+0.09062293
^{57}Co	7/2	+4.733
^{93}Nb	9/2	+6.1705

Electromagnetic moments

- The nucleus has charge (monopole moment).
- No dipole moment since it is all positive.
- But if the nucleus is not spherically symmetric, it will have a quadrupole moment.



Classical moments

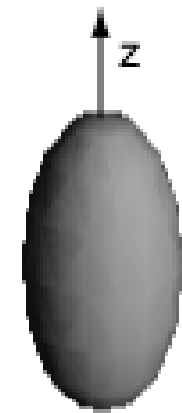


Electric Quadrupole Moment

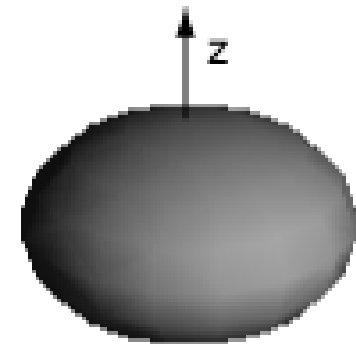
- For a point charge e : $eQ = e(3z^2 - r^2)$.
- Spherical symmetry $\blacktriangleright x^2 = y^2 = z^2 = r^2/3 \blacktriangleright Q = 0$.
- For a proton:

$$eQ = e \int \psi^* (3z^2 - r^2) \psi dv$$

- In the xy-plane: $Q \sim -\langle r^2 \rangle$.
- $\langle r^2 \rangle$ is the mean square radius of the orbit.
- Along z: $Q \sim +2 \langle r^2 \rangle$.
- Expected maximum $\sim e r_0^2 A^{2/3}$.
- *Up to $50 \times 10^{-30} \text{ em}^2$.*
- *Up to 0.5 eb .*



$Q > 0$
Prolate



$Q < 0$
Oblate

Electric Quadrupole Moment

- Closed shell ► Spherically symmetric core.
- Test for shell model
- Strongly deformed nuclei.....!

Nuclide	Q (b)
${}^2\text{H}$ (D)	+0.00288
${}^{17}\text{O}$	-0.02578
${}^{59}\text{Co}$	+0.40
${}^{63}\text{Cu}$	-0.209
${}^{133}\text{Cs}$	-0.003
${}^{161}\text{Dy}$	+2.4
${}^{176}\text{Lu}$	+8.0
${}^{209}\text{Bi}$	-0.37